

Conferința
Diaspora în
Cercetarea
Științifică
Românească
București, 17-19 Septembrie 2009

EFECTUL CONFINĂRII CUANTICE ASUPRA STRUCTURII ENERGETICE A SISTEMELOR CU DIMENSIONALITATE REDUSĂ

Magdalena Lidia Ciurea

National Institute of Materials Physics, Bucharest-Magurele



NATIONAL INSTITUTE OF MATERIALS PHYSICS BUCHAREST-MAGURELE



CONTENT:

1. INTRODUCTION
2. 2D SYSTEMS
3. 1D SYSTEMS
4. 0D SYSTEMS
5. CONCLUSIONS





1. INTRODUCTION

**Low dimensional system (LDS) \Leftrightarrow
nanometric size on *at least one* direction.**

**Ratio between the number of atoms located at the
surface/interface and the total number of atoms:**

$$N_S^{(\delta)} / N = 2(3 - \delta)a / d_\delta$$

δ – system dimensionality; a – interatomic distance; d_δ – (minimum) LDS size;
 $N_S^{(\delta)} / N = 0.5$, $a \approx 0.25$ nm $\Rightarrow d_0 = 3$ nm, $d_1 = 2$ nm, $d_2 = 1$ nm.



**Surface/interface \Leftrightarrow potential barrier \Leftrightarrow
wall of a quantum well \Rightarrow quantum confinement (QC)**

- **QC – ZERO ORDER EFFECT**
nature of the material – first order effect
➔ ***infinite* quantum well = first approximation**

- **Shape of the quantum well \Rightarrow ratios between energies
of consecutive levels \Leftrightarrow choice of the shape**
➔ ***rectangular* quantum well = good approximation**





2. 2D SYSTEMS

Plane nanolayers

Hamiltonian splitting (exact):

- parallel part – Bloch-type \Rightarrow 2D band structure;
- perpendicular part – infinite rectangular quantum well (IRQW) \Rightarrow QC levels

$$\rightarrow \varepsilon^{(2)} = \varepsilon_n(k_x, k_y) + \frac{2\pi^2 \hbar^2}{m_{\perp}^* d^2} p^2, \quad p > 0.$$



$T = 0 \text{ K} \Rightarrow \varepsilon_n(k_x, k_y) = E_n, E_{\text{QC}0} = ?$ **By convention, $E_{\text{QC}0} \equiv 0$.**

$$\begin{aligned} \rightarrow \varepsilon^{(2)} &= \left[\varepsilon_n(k_x, k_y) + \frac{2\pi^2 \hbar^2}{m_{\perp}^* d^2} \right] + \frac{2\pi^2 \hbar^2}{m_{\perp}^* d^2} (p^2 - 1) \\ &\equiv \varepsilon_n^{(s)}(k_x, k_y) + \varepsilon_{p-1}. \end{aligned}$$

QC levels located in the band gap!





➤ **Application: quantum well solar cells (QWSC)**

Matrix element of electric dipole interaction Hamiltonian:

$$H_{fi} = -\frac{2}{d} e \int_0^l \vec{E} \cdot \vec{r} \sin \frac{p_f \pi z}{d} \sin \frac{p_i \pi z}{d} dz \neq 0$$

$$\vec{E} = \vec{E}_{\parallel} \Rightarrow p_f = p_i; \quad \vec{E} = \vec{E}_{\perp} \Rightarrow p_f - p_i = 2p - 1$$

$$p_i = 1 \Rightarrow \text{internal quantum efficiency: } \eta_{\perp} = \frac{4096 e^2}{\pi^2 c \hbar \epsilon_0 \epsilon_r^2} \cdot \frac{p^2 \sin^2 i}{(4p^2 - 1)^4}$$



3. 1D SYSTEMS

Cylindrical nanowires

Hamiltonian splitting (approximate):

- longitudinal part – Bloch-type \Rightarrow 1D band structure;
- transversal part – infinite rectangular quantum well (IRQW) \Rightarrow QC levels

$$\rightarrow \epsilon^{(1)} = \epsilon_n(k_z) + \frac{2\pi^2 \hbar^2}{m_i^* d^2} x_{p,l}^2$$

$x_{p,l}$ – p -th non-null zero of cylindrical Bessel function $J_l(x)$





$T = 0 \text{ K} \Rightarrow \varepsilon_n(k_z) = E_v, E_{\text{QC0}} = ? \Rightarrow E_{\text{QC0}} \equiv 0.$

$$\begin{aligned} \rightarrow \varepsilon^{(1)} &= \left[\varepsilon_n(k_z) + \frac{2\pi^2 \hbar^2}{m_t^* d^2} x_{1,0}^2 \right] + \frac{2\pi^2 \hbar^2}{m_t^* d^2} (x_{p,l}^2 - x_{1,0}^2) \\ &\equiv \varepsilon_n^{(s)}(k_z) + \varepsilon_{p-1,l}, l - \text{orbital quantum number.} \end{aligned}$$

QC levels located in the band gap!

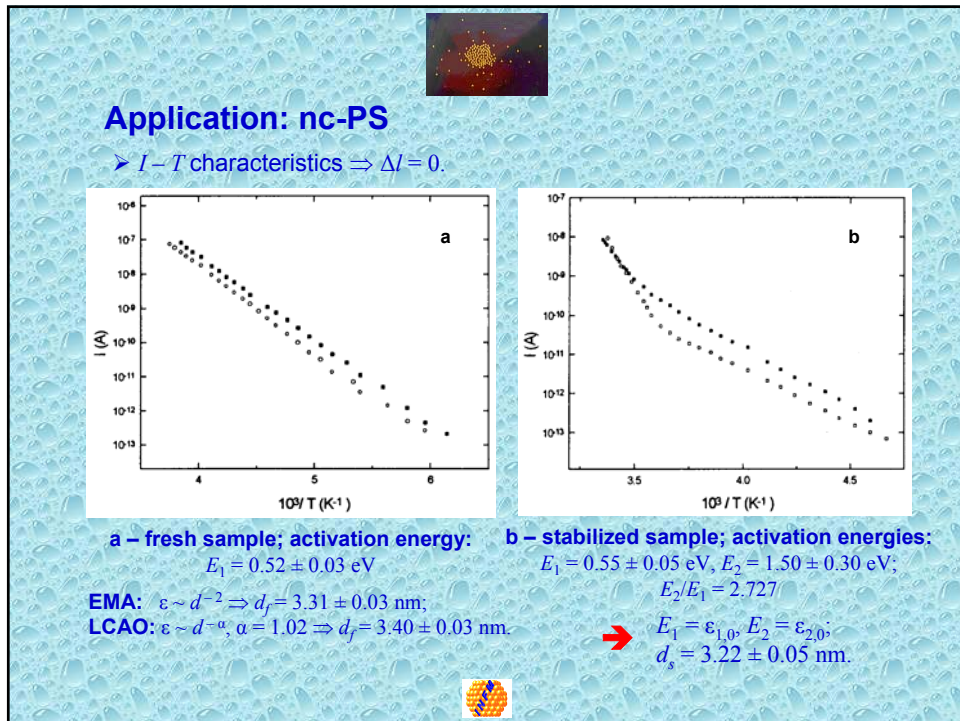
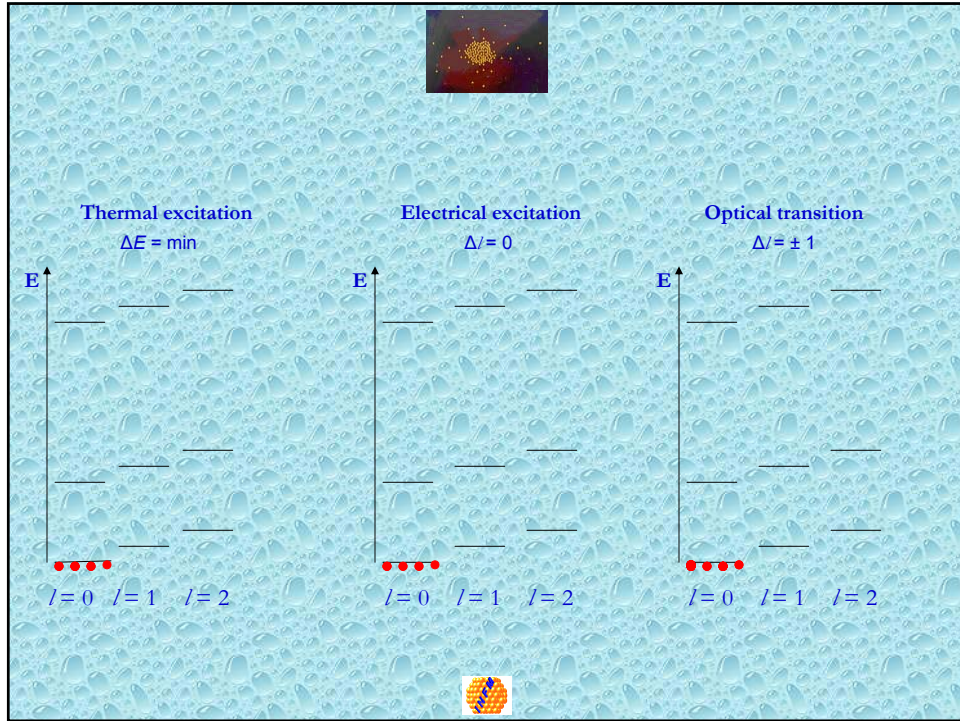


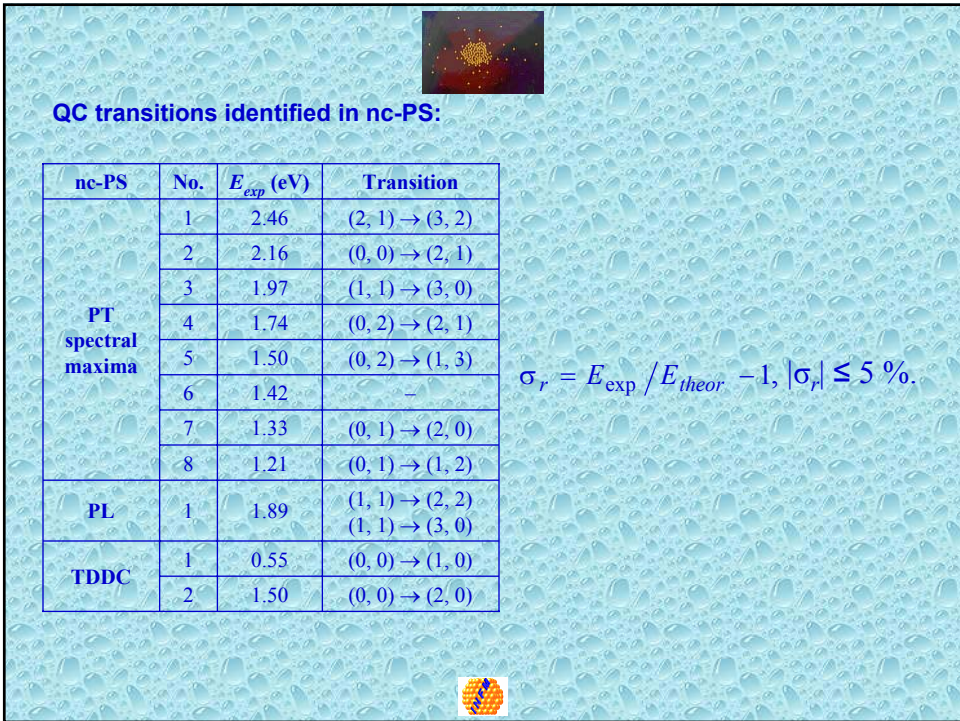
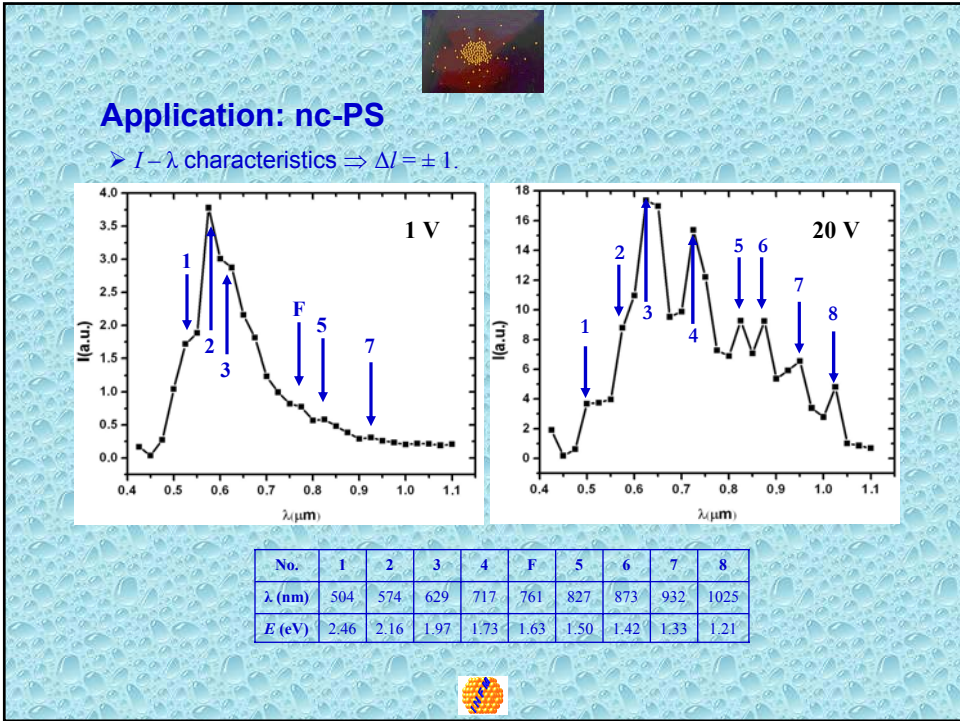
**Valence band = particle reservoir \Rightarrow
excitation transitions start from the fundamental level
 \rightarrow activation energy ratio**

$$R_{nw} = \frac{x_{p'+1,l''}^2 - x_{1,0}^2}{x_{p'+1,l'}^2 - x_{1,0}^2}$$

**Thermal transition $\Leftrightarrow \Delta\varepsilon = \text{minimum};$
Electrical transition ($eU \gg k_B T$) $\Leftrightarrow \Delta l = 0;$
Optical transition $\Leftrightarrow \Delta l = \pm 1.$**









4. 0D SYSTEMS

Spherical nanodots – quantum dots

➤ $d \leq 5$ nm \Rightarrow no more bands \Leftrightarrow groups of energy levels = quasibands

$$\rightarrow \varepsilon^{(0)} = \frac{2\pi^2 \hbar^2}{m^* d^2} x_{p,l}^2$$

$x_{p,l}$ – p -th non-null zero of spherical Bessel function $j_l(x)$



$$T = 0 \text{ K} \Rightarrow \varepsilon^{(0)} = E_v; E_{\text{QC0}} = ? \Rightarrow E_{\text{QC0}} \equiv 0.$$

$$\rightarrow \varepsilon^{(0)} = \frac{2\pi^2 \hbar^2}{m^* d^2} x_{1,0}^2 + \frac{2\pi^2 \hbar^2}{m^* d^2} (x_{p,l}^2 - x_{1,0}^2) \equiv E_V + \varepsilon_{p-1,l}$$

l – orbital quantum number.

QC levels located in the quasiband gap!





EMA: $\epsilon \sim d^{-2}$

LCAO: $\epsilon \sim d^{-\alpha}$, $\alpha = 1.39$

$$\rightarrow m^* \approx m_{\infty}^* + \frac{a}{d} (m_{0e} - m_{\infty}^*)$$

$\rightarrow m^* \approx m_{0e}$ for quantum dots



No more proper VB = no more particle reservoir
 \Rightarrow excitation transitions: from the last occupied level to the next one \rightarrow selection rules

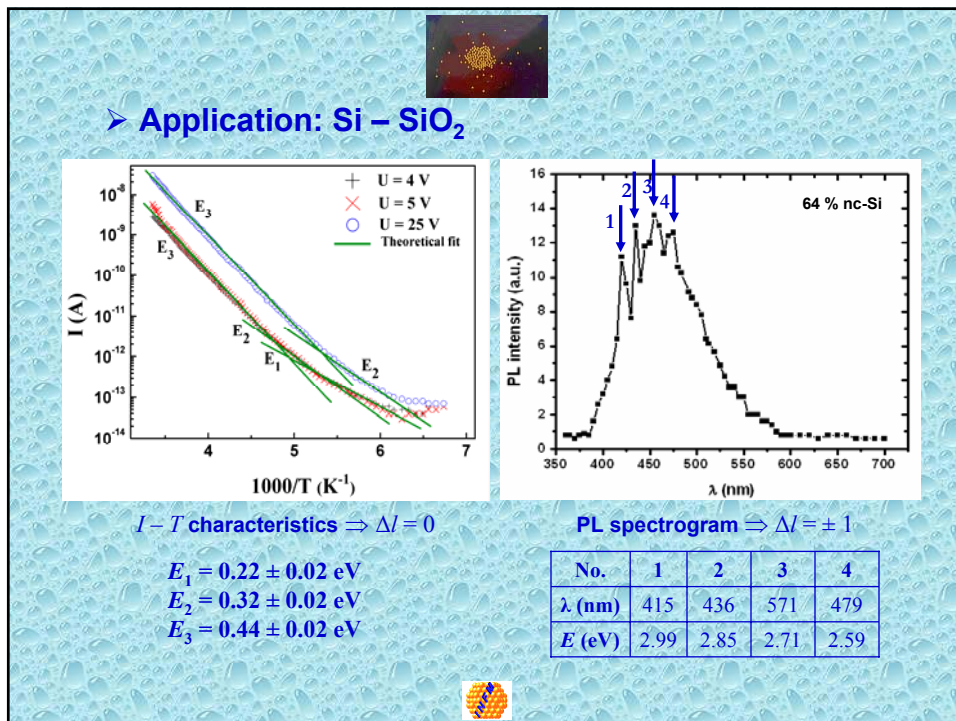
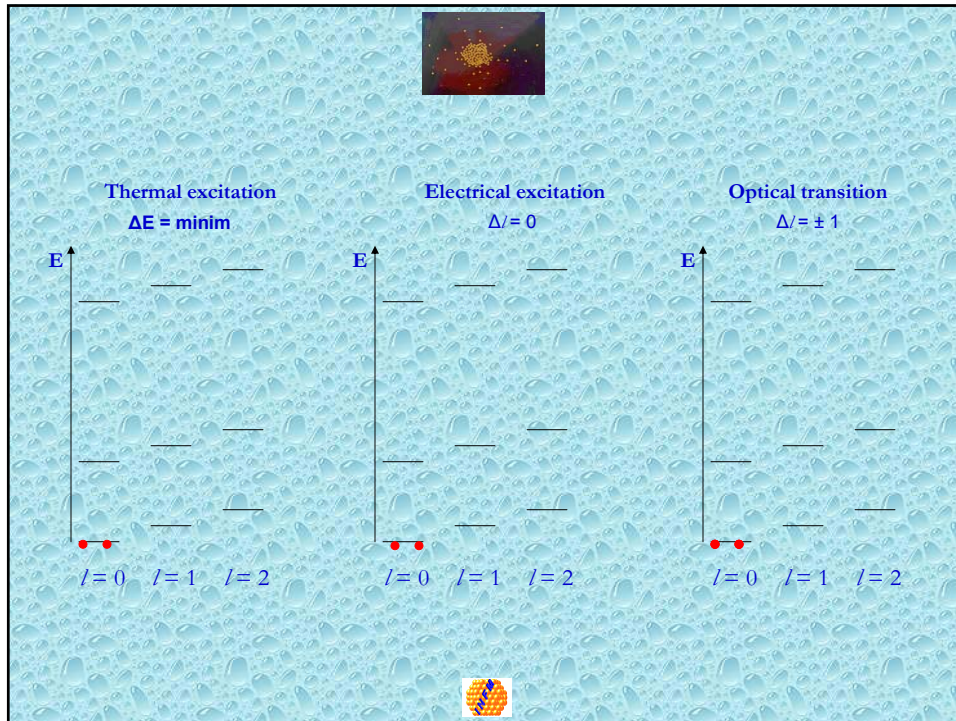
$$\rightarrow R_{qd} = \frac{x_{l', p'+1}^2 - x_{l', p+1}^2}{x_{l', p'+1}^2 - x_{l, p+1}^2}$$

Thermal transition $\Leftrightarrow \Delta\epsilon = \text{minimum}$;

Electrical transition ($eU \gg k_B T$) $\Leftrightarrow \Delta l = 0$;

Optical transition $\Leftrightarrow \Delta l = \pm 1$.







QC transitions identified in Si – SiO₂

Si – SiO ₂	No.	E_{exp} (eV)	Transition
PL ($d = 4,92$ nm)	1	2,99	(1, 2) → (6, 1)
	2	2,85	(0, 1) → (5, 2)
	3	2,71	(1, 1) → (6, 0)
	4	2,59	(1, 1) → (5, 2)
TDDC ($d = 5,28$ nm)	1	0,22	(0, 1) → (1, 1)
	2	0,32	(1, 1) → (2, 1)
	3	0,44	(2, 1) → (3, 1)

$$|\sigma_r| < 3 \%$$



5. CONCLUSIONS

- Almost **ALL** the energies measured in electrical transport, phototransport and photoluminescence \leftrightarrow transitions between QC levels.
Different quantum selection rules \longrightarrow different energy levels.
- Differences between model and experiment due to:
 - ✓ size distribution;
 - ✓ shape distribution;
 - ✓ finite depth of the quantum well.



