

# High-harmonic generation in small molecules: influence of vibration and laser dressing

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# Outline

- Introduction.
- Harmonic generation in molecules:
  - molecular vibrational motion;
  - laser dressing.
- Results.

*Atomic units are used.*

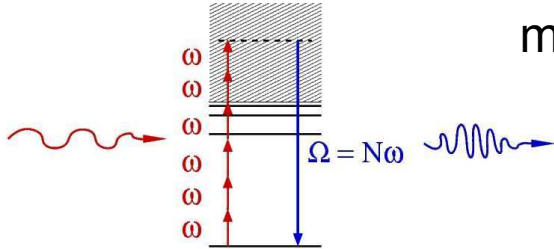
# Motivation

- To be able to describe accurately the harmonic generation in molecular systems (taking into account the effect of the molecular vibration, laser dressing, etc...).
- To be able to understand in simple physical terms the relevant aspects of the harmonic generation dynamics, opening the way to its control.
- Availability of new infrared laser sources ( $\sim 1\text{-}2\ \mu\text{m}$ ), e.g. Advanced Laser Light Source (ALLS) in Canada.

## Introduction

Harmonic Generation -  
definition and applications.

Harmonic generation: an atom/molecule irradiated by a laser with frequency  $\omega$  emits radiation at frequencies  $N\omega$ , multiples of the laser frequency.



In contrast to atoms, in molecules **additional effects** influence the HG process:

- ✓ **the interference effect**: the harmonic radiation emitted from each of the atomic sites interfere;
- ✓ the effect of **nuclear vibration**, greater importance of **laser dressing**, ...

Applications of harmonic generation:

- generate extremely short (attosecond) light pulses;
- coherent X-ray spectroscopy;
- imaging molecular orbitals (molecular tomography);
- resolve molecular dynamics.

# The single-atom/molecule dipole moment (SFA)

STRONG FIELD APPROXIMATION<sup>1</sup>:

**NEW<sup>2</sup>: Insert here the propagator from  $t'$  to  $t$  for the nuclear motion in the molecular ion (1, or 2 BO-potentials with laser coupling)**

Dipole momentum:  $P(t) = -2 \text{Im}$

$$\int_0^t dt' \langle \text{Initial molec. state}(t) | -i\vec{\nabla} | \text{Volkov propagator}(t,t') \times \text{Interaction Hamiltonian}(t') | \text{Initial molecular state}(t') \rangle$$

**recombination** matrix element

the **free-electron propagator** in the laser field

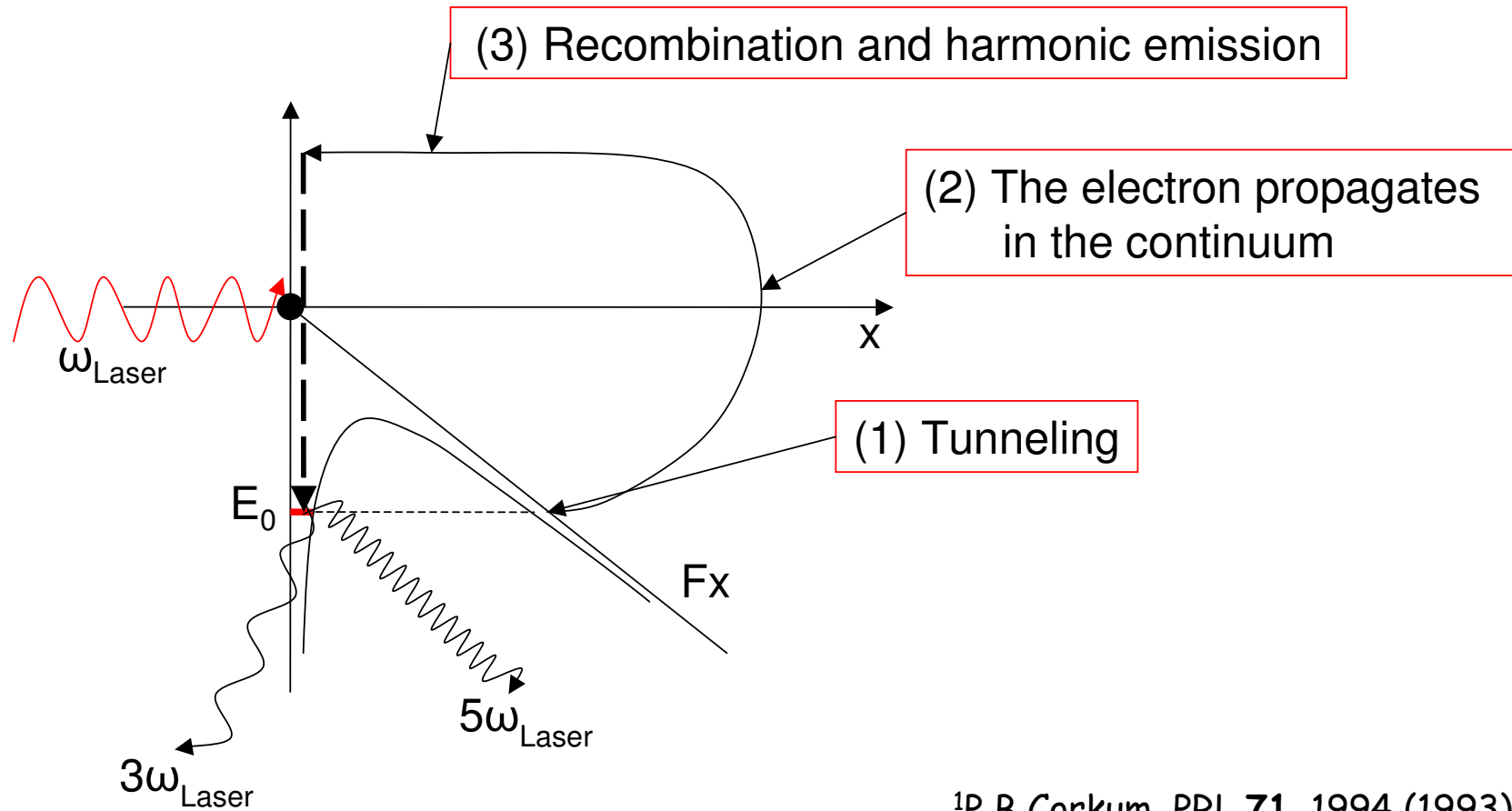
**ionization** matrix element

<sup>1</sup>Lewenstein, Balcou, Ivanov, L'Huillier and Corkum;

<sup>2</sup>C C Chirilă and M Lein, J. Phys. B **39**, S437 (2006).

# The simple-man's (3-step) model<sup>1</sup>

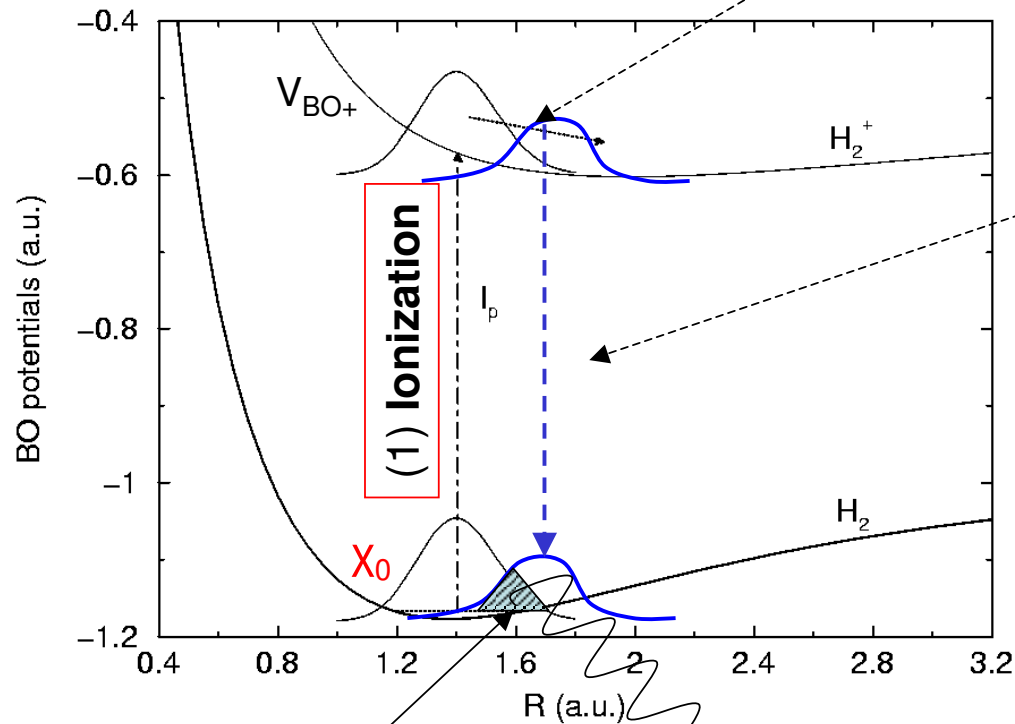
**HG** can be understood as a *sequence* of the following *three steps* for the electron:



<sup>1</sup>P B Corkum, PRL **71**, 1994 (1993)

# The effect of vibration on HG: motion in one level (1L)

(2) **Field acceleration** of the active electron & evolution of the vibrational wave packet in the BO potential of the molecular ion



Vibrational autocorrelation function  $C(\Delta)$

(3) **Recombination**

Autocorrelation function:

$$C(\Delta) = \int_0^{\infty} dR \chi_0(R) \exp(-i \hat{H}_R \Delta) \chi_0(R),$$

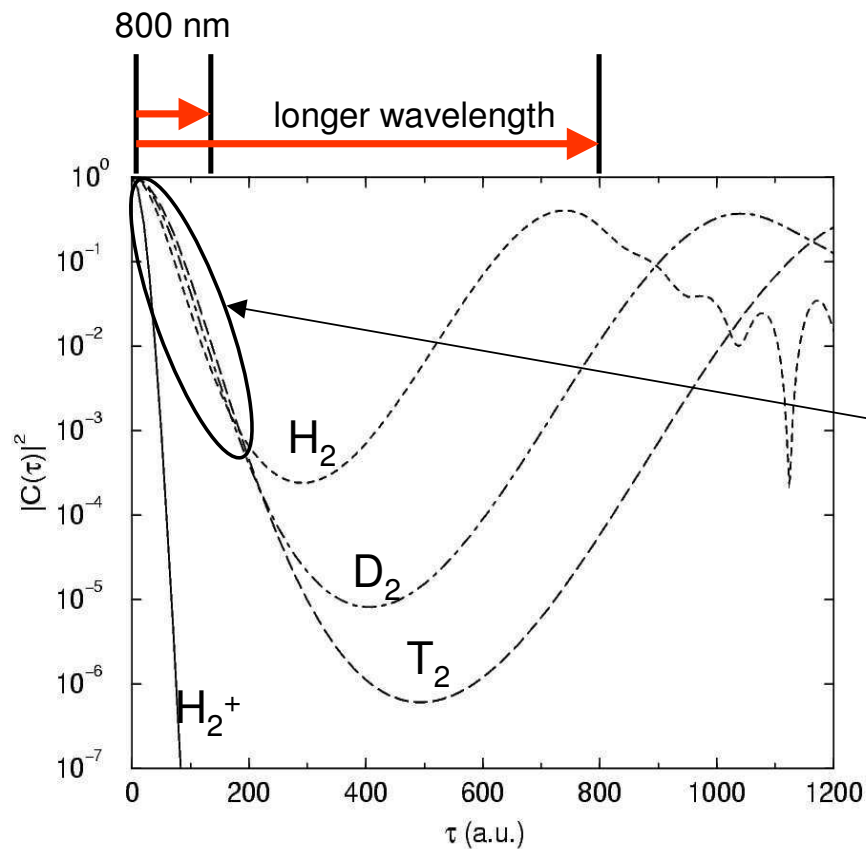
where  $\chi_0(R)$  is the vibrational part of the molecular ground-state and  $\hat{H}_R$  is the vibrational Hamiltonian of the molecular ion (*after* ionization):

$$\hat{H}_R = -1/M \partial^2/\partial R^2 + V_{BO+}(R).$$

(M is the mass of one nucleus.)



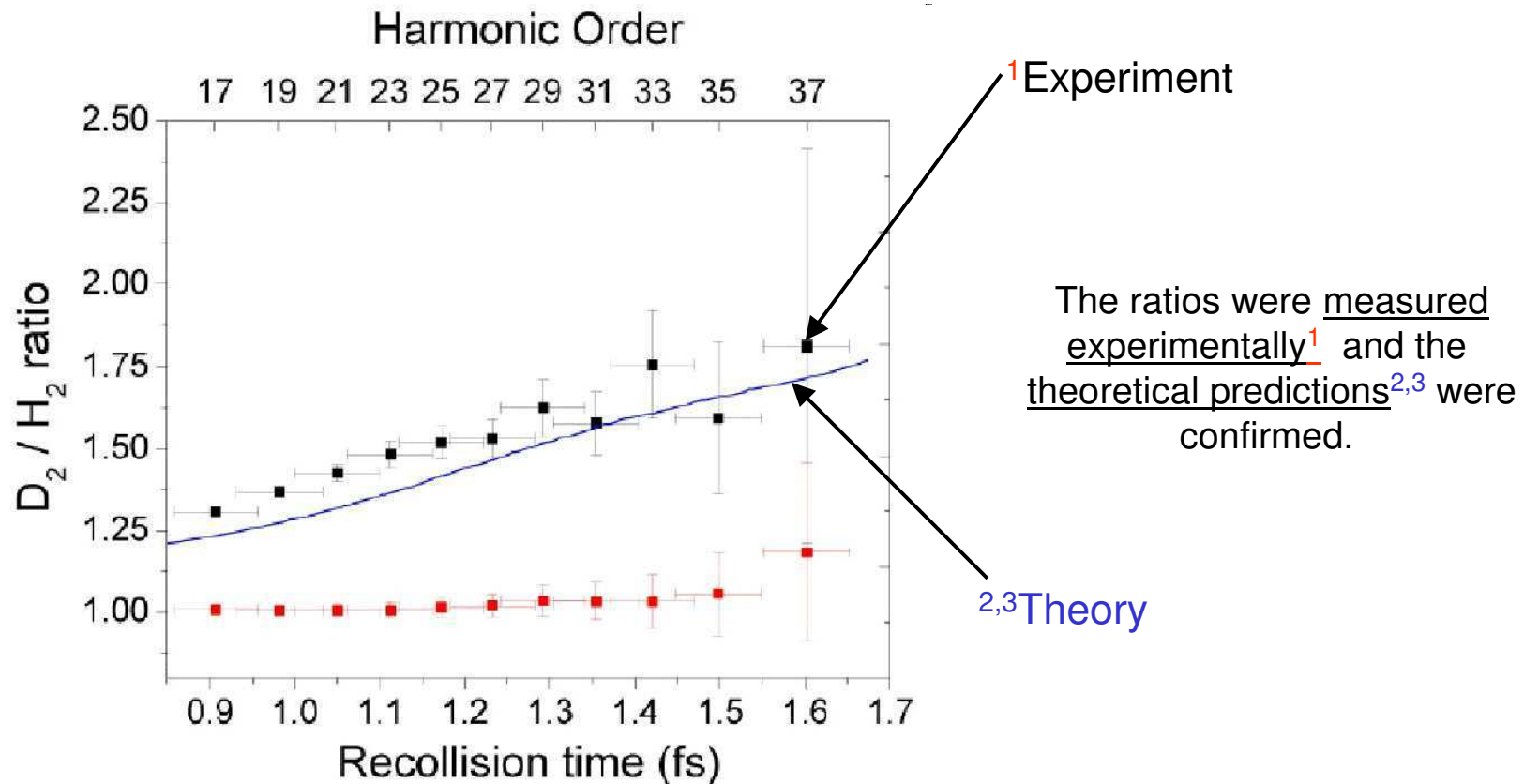
# The autocorrelation function (1L): the isotope effect



The effect of vibration is smaller for heavier isotopes!

The vibrational autocorrelation function for different molecular isotopes.

# Ratio $D_2/H_2$ of HG yields: experiment vs. theory



<sup>1</sup>S Baker, J Robinson, C A Haworth, H Teng, R A Smith, C C Chirilă, M Lein, J W G Tisch, and J P Marangos, *Science* **312**, 424 (2006);

<sup>2</sup>M Lein, *PRL* **94**, 053004 (2005);

<sup>3</sup>C C Chirilă and M Lein, *J. Phys. B* **39**, S437 (2006).

# The molecular dipole momentum (SFA)

The electronic dipole momentum along the laser polarization direction reads:

$$P_x = -4 \operatorname{Im} \int_0^t dt' \int_0^\infty dR \chi_0(R) \left[ \frac{2\pi}{i(t-t') + 1} \right]^{3/2} \exp[-i S(t', t)] \times$$

$$\langle \Psi_0(\vec{r}_1, \vec{r}_2) | \hat{P}_{x1} | \text{Volkov}(t, \vec{r}_1) \rangle \times [ \langle \Psi_g^+(\vec{r}_2) |, \langle \Psi_u^+(\vec{r}_2) | ] \times$$

$$\hat{U}_R(t, t') \begin{bmatrix} \langle \Psi_g^+(\vec{r}_2) | \\ \langle \Psi_u^+(\vec{r}_2) | \end{bmatrix} \text{Volkov}(t', \vec{r}_1) | E(t') x_1 | \Psi_0(\vec{r}_1, \vec{r}_2) \rangle \chi_0(R)$$

$$i \frac{d}{dt} \begin{bmatrix} \chi_g(R, t) \\ \chi_u(R, t) \end{bmatrix} = \begin{pmatrix} \hat{H}_{0g}^+ & \vec{E}(t) \cdot \vec{D}(R) \\ \vec{E}(t) \cdot \vec{D}(R) & \hat{H}_{0u}^+ \end{pmatrix} \begin{bmatrix} \chi_g(R, t) \\ \chi_u(R, t) \end{bmatrix}$$

ground vibrational state of the molecule

semiclassical action

first two electronic states of the molecular ion

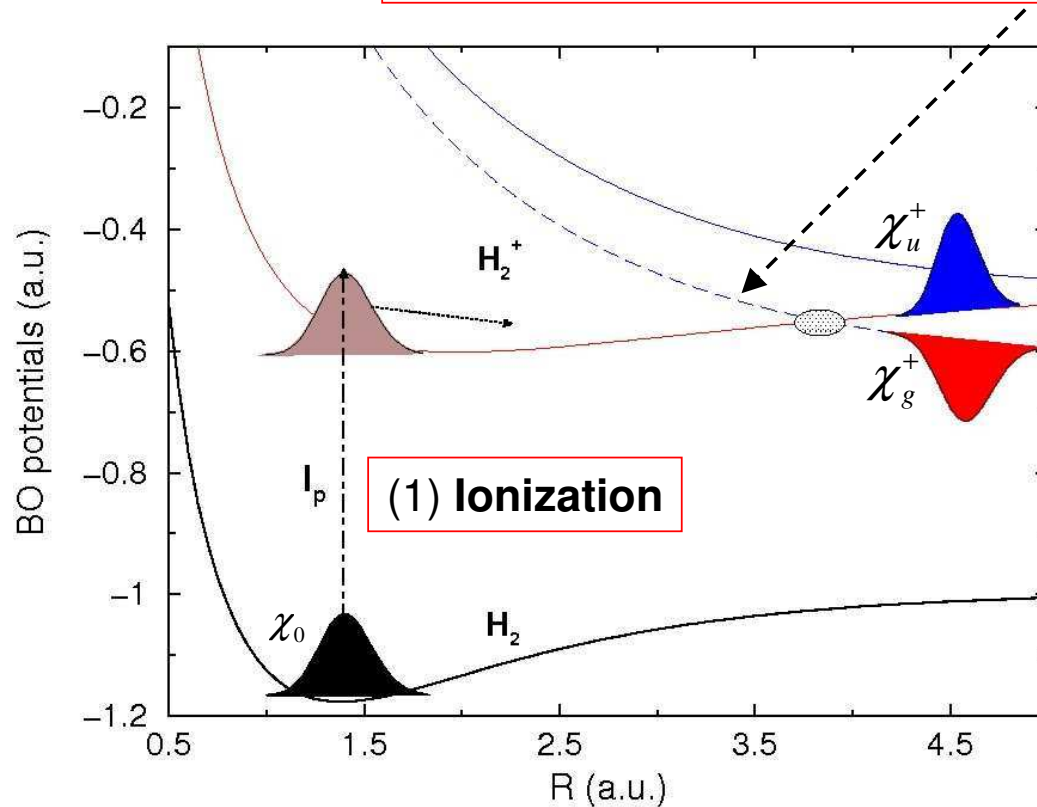
the unbound active electron

ground electronic state of the molecule

dipole coupling between the BO levels of the molecular ion

# The effect of vibration on HG: motion in two laser-dressed levels (2L)

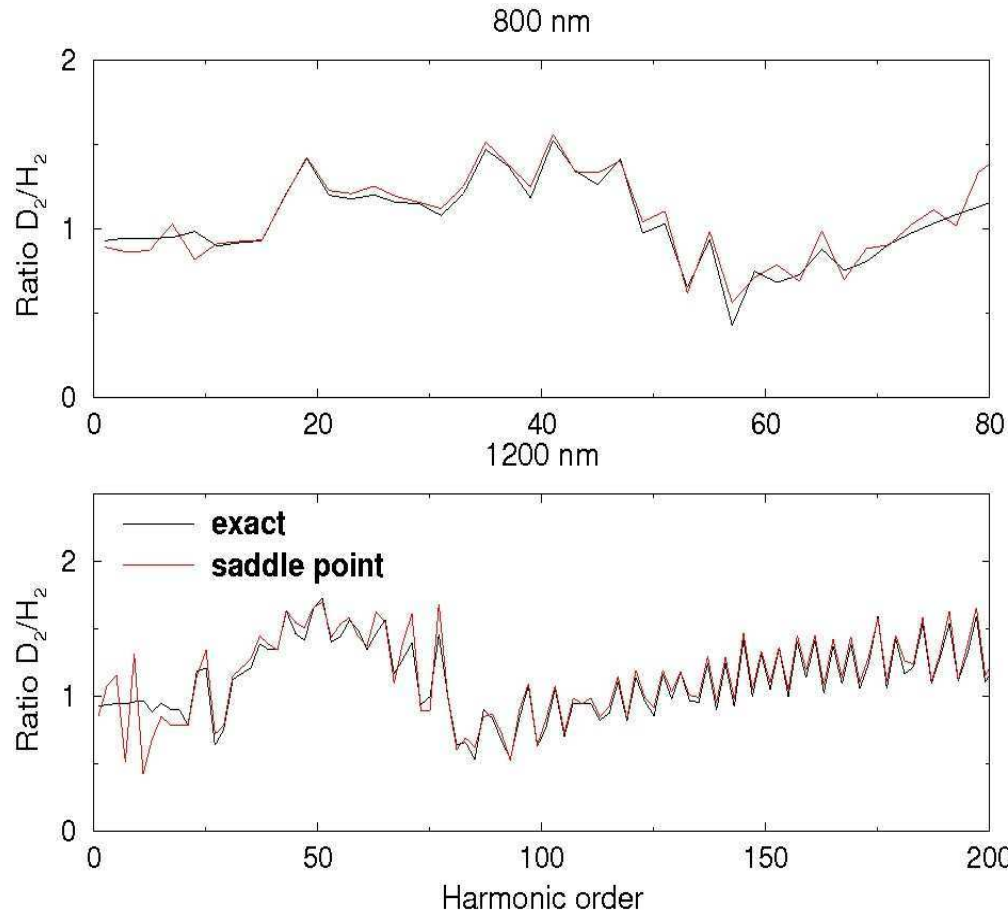
(2) **Field acceleration** of the active electron & evolution of the vibrational wave packet in the BO potential of the molecular ion



✓ After ionization, the vibrational wave packet is launched on the two lowest-laying Born-Oppenheimer potentials in the molecular ion.

✓ These two lowest-laying Born-Oppenheimer potentials in the molecular ion are dressed by the laser field.

# Numerical aspects: the saddle-point method\*



Ratio of the harmonic intensities in  $H_2$  and  $D_2$ , aligned parallel ( $\theta = 0$ ) to the laser electric field.

The laser intensity is  $5 \times 10^{14} \text{ W/cm}^2$ , the pulse is trapezoidal, with four optical cycles of constant amplitude.

Typical execution times:

- (1) exact SFA calculation  $\approx$  1 week;
- (2) saddle point method  $\approx$  1 day.

\*saddle point approximation for the integration over the ionization time.

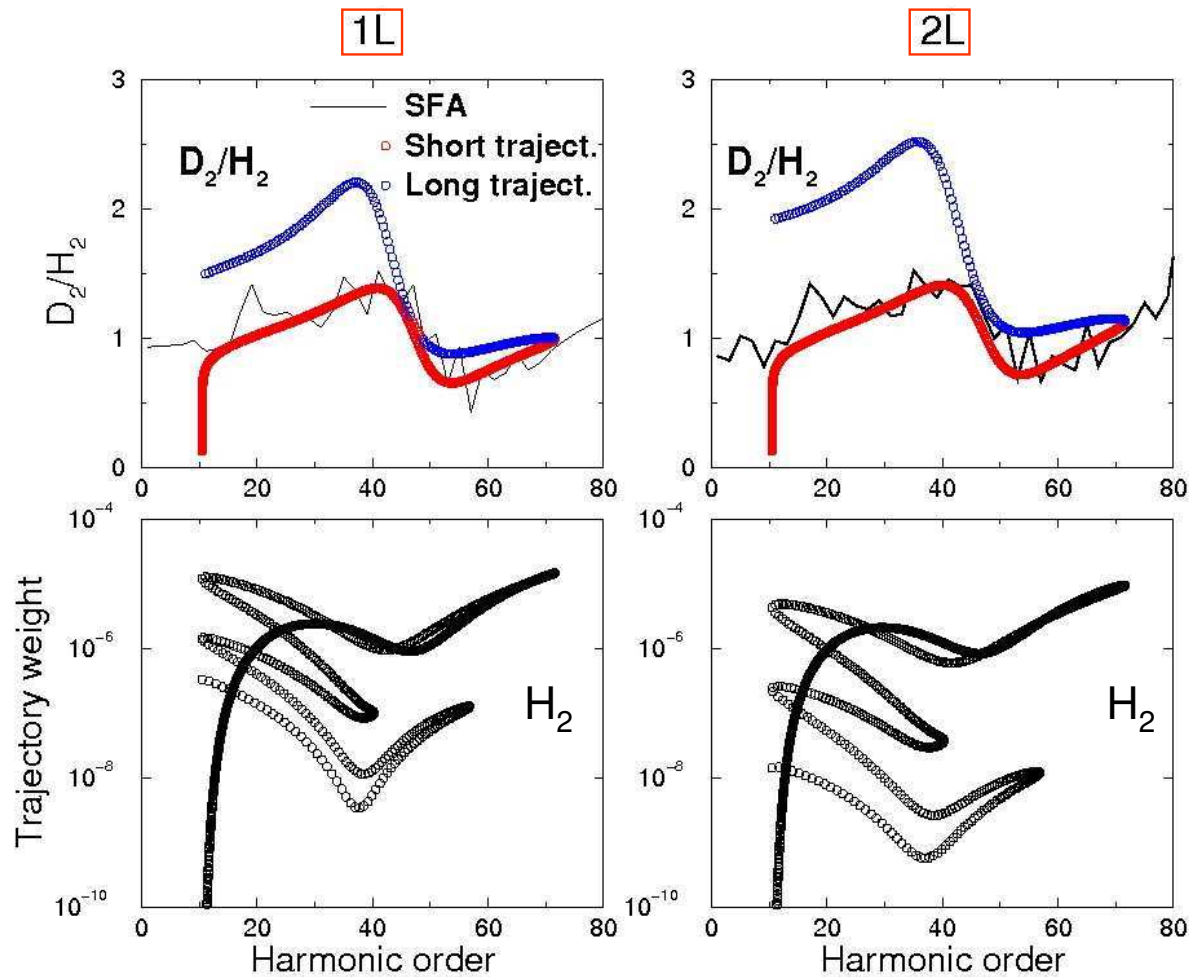
## Simple-man's model: weight of an electronic trajectory

Given a certain **classical electronic trajectory**, that starts with the electron tunnelling out at time  $t'$  and recombination taking place at time  $t$ , the weight of the trajectory (*i.e.* the contribution to the total harmonic yield) is:

$$\propto \exp[-2/3(2I_p)^{3/2} / |E(t')|] \left| \int_0^\infty dR \cos(k_{ret} R \cos \theta / 2) [\chi_0(R), 0] \hat{U}_R(t, t') \begin{bmatrix} \chi_0(R) \\ 0 \end{bmatrix} \right|^2$$

where  $k_{ret}$  is the returning velocity, and  $\theta$  is the angle between the molecular axis and the laser polarization axis.

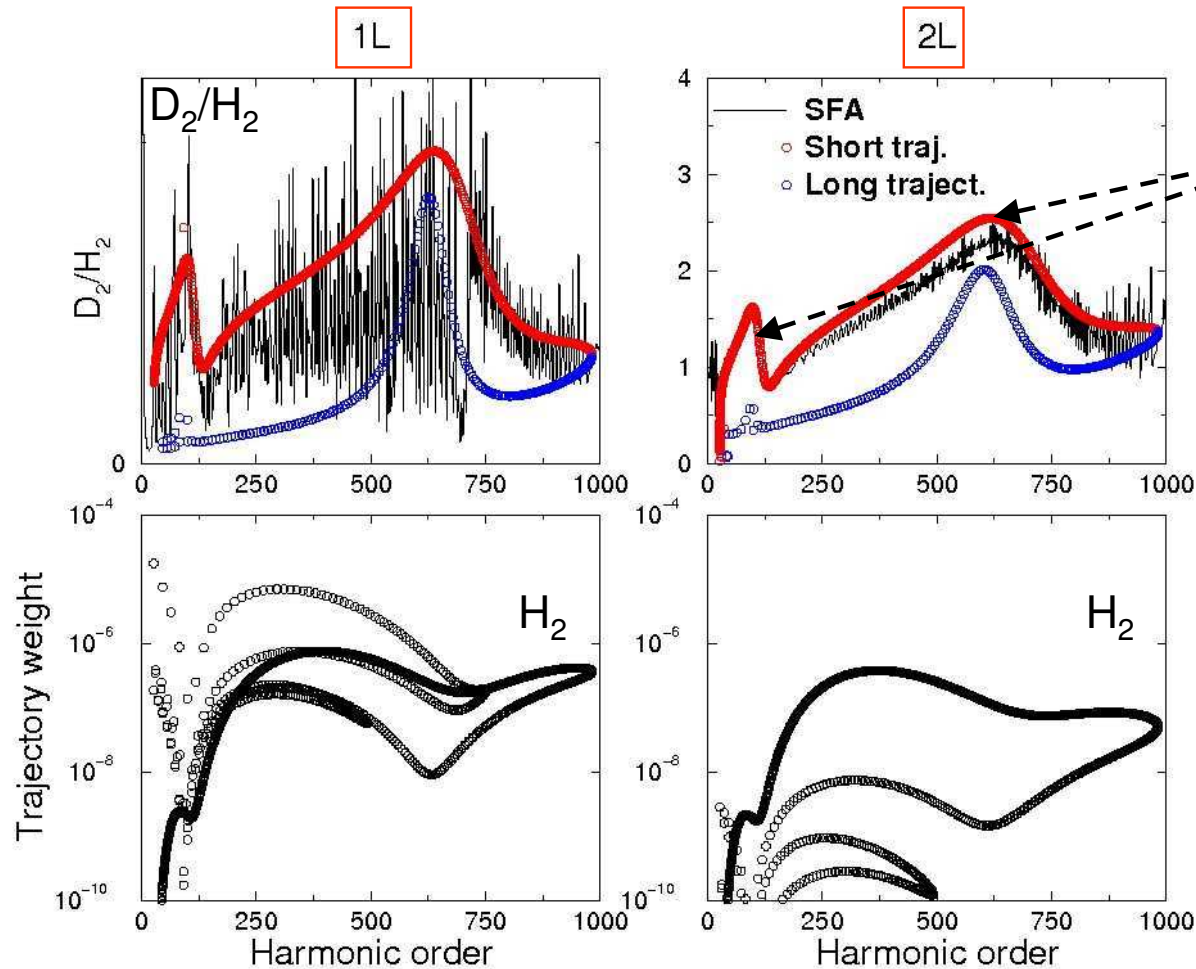
# Results (800 nm)



The results are hardly affected by the field dressing.

\*  $\theta = 0$ , laser intensity:  $I = 5 \times 10^{14} \text{ W/cm}^2$ .

# Results (2000 nm)



Signatures of the two-center interference !

The results are significantly affected by the field dressing.

The dressing strongly affects the long electronic trajectories.

Field dressing selects short trajectories!

\*  $\theta = 0$ , laser intensity:  $I = 5 \times 10^{14} \text{ W/cm}^2$ .



## Conclusions

- The **strong-field approximation** can be extended to take into account the **effects of the molecular vibration** on the harmonic emission.
- The laser dressing of the BO-potential surfaces causes non-negligible changes in the harmonic emission.
- The effect of the field coupling in the molecular ion increases with the increase of the laser wavelength.

Thank you for your attention !