Calculation of the energies of hard X-rays generated by the interaction between relativistic electrons and very intense laser beams

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1) The calculation method in the case of the relativistic Thomson scattering; interaction between ultraintense laser beams ($I>10^{19}W/cm^2$ and $a>10$) and electrons whose initial velocities are small.

- A. Popa, "Accurate calculation of high harmonics generated by relativistic Thomson scattering," J. Phys. B: At. Mol. Opt. Phys., 41, 015601, 2008.

2) Calculation of the energies of hard X rays generated by the interaction between relativistic electrons (20-100 MeV) and very intense laser beams.

3) A brief presentation of the final results of the GRANT CNCSIS entitled "NEW CONNECTION BETWEEN QUANTUM AND CLASSICAL EQUATIONS WITH APPLICATIONS TO THE MODELING OF THE STATIONARY STATES AND TO THE INTERACTIONS BETWEEN ULTRAINTENSE ELECTROMAGNETIC FIELDS AND ATOMS."

- A. Popa, J. Phys. A: Mathematical and General, 36, 7569, 2003.
- A. Popa, J. Of Chemical Physics, 122, 244701, 2005.
- A. Popa, IEEE J. Quantum Elect., 40, 1519, 2004.
- A. Popa, IEEE J. Quantum Elect., 43, 1183, 2007.
- A. Popa, European Physical Journal D, 49, 2008, in print.

1) **High harmonics generated by relativistic Thomson scattering at the interaction between ultraintense e.m. linear polarized field (a>10) and electrons whose initial velocities are small**

First stage: solution of the relativistic equations of motion

$$\frac{d}{dt}(\gamma\beta_x) = -a\omega(1-\beta_z)\cos\eta$$
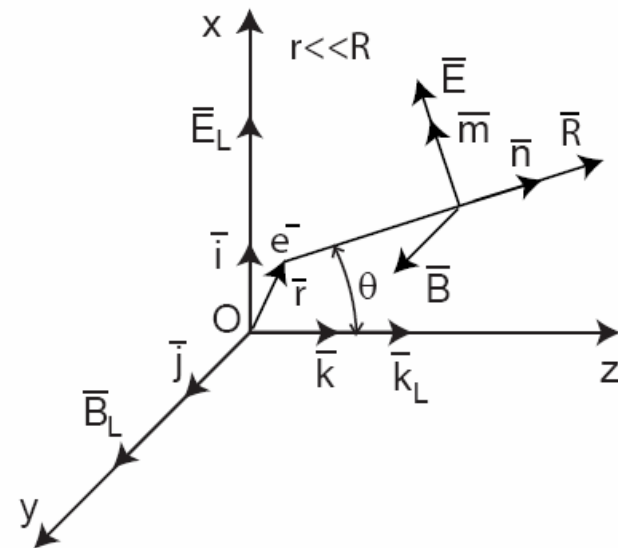
$$\frac{d}{dt}(\gamma\beta_z) = -a\omega\beta_x\cos\eta$$

$$E_L = E_M\cos\eta \quad B_L = B_M\cos\eta$$

$$\gamma = \left(1-\beta_x^2-\beta_z^2\right)^{-\frac{1}{2}} \quad a = \frac{eE_M}{mc\,\omega}$$

$$\beta_x = v_x/c \quad \beta_z = v_z/c \quad \eta = \omega t - k_L z$$

Result: $\beta_x \quad \beta_z \quad \dot{\beta}_x \quad \dot{\beta}_z$

Remarkable property of the solutions: are functions only of η
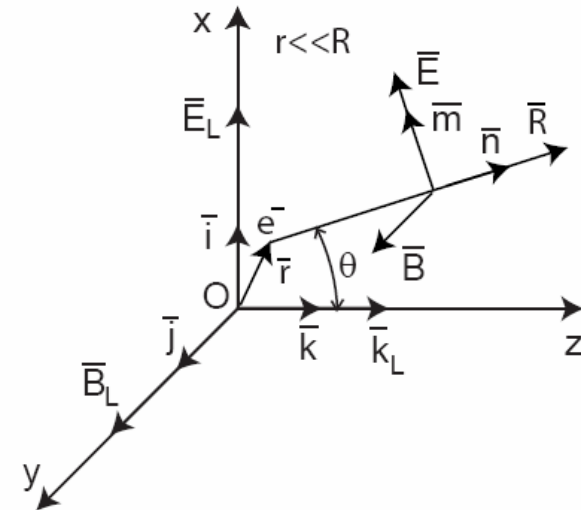
Second stage : calculation of the field generated by the motion of the electron with the Lienard Wiechert relation.

$$\overline{E} = \frac{-e}{4\pi\varepsilon_0 cR} \cdot \frac{\overline{n} \times \left[\left(\overline{n} - \overline{\beta}\right) \times \dot{\overline{\beta}}\right]}{\left(1 - \overline{n} \cdot \overline{\beta}\right)^3}$$

$$\overline{E} = -E_M \frac{r_0}{R} f(\eta,\theta) \cdot \overline{m} \qquad \overline{m} \perp \overline{n}$$

$$f(\eta,\theta) = \frac{\cos\theta + \dfrac{1}{2}a^2 \sin^2\eta\left(1 - \cos\theta\right) + a\sin\eta\sin\theta}{\left[1 + \dfrac{1}{2}a^2 \sin^2\eta\left(1 - \cos\theta\right) + a\sin\eta\sin\theta\right]^3}$$
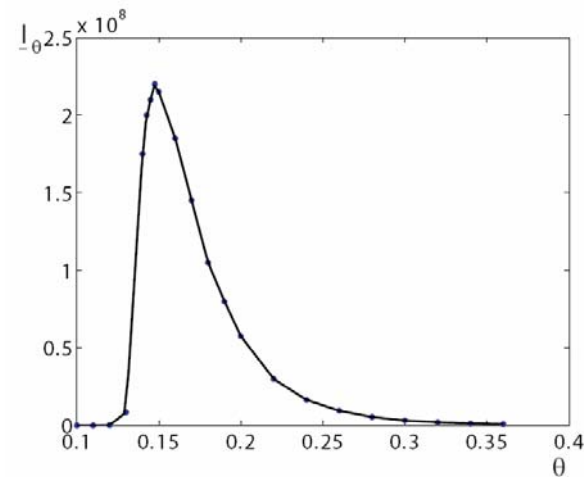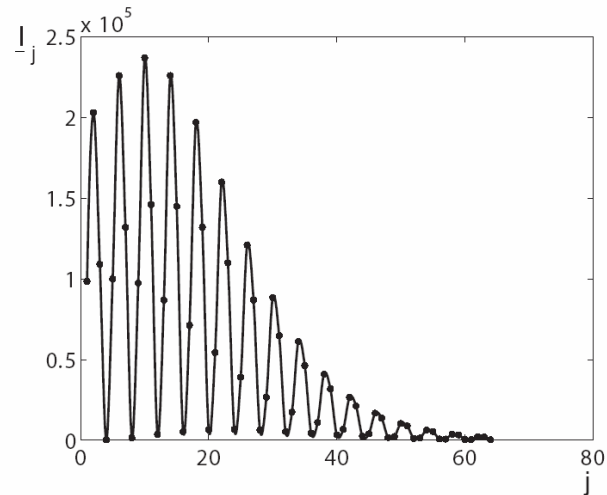
The main property : E periodic in η

Third stage: calculations of the harmonics of the field and of the spectrum of the intensity of the Thomson scattered beam.

$$\overline{E} = \sum_{j=1}^{\infty} \overline{E}_j \qquad \overline{B} = \sum_{j=1}^{\infty} \overline{B}_j \qquad \overline{B}_j = \overline{n} \times \left|\frac{\overline{E}_j}{c}\right| \qquad I_j = I_L \frac{r_0^2}{R^2} N_e f_j^2 \qquad \begin{array}{l} k_j = jk_L \\ \\ \omega_j = j\omega_L \end{array}$$

Discrete spectrum for a=15 and θ = 0.1874 radians
- Angular distribution in the polarization plane for a=15

Results in agreement to the experiment reported in:
Phys. Rev. Lett. 91, 195001, 2008, Laboratoire d'Optique Appliquee, ENSTA

2) Calculation of the energies of hard X rays generated by the interaction between relativistic electrons and very intense laser beams.

Experimental data in this moment for interactions between:
- Laser beam for intensities 10^{16}-10^{18}W/cm^2
- Relativistic electrons of energies 20-100 MeV

Generation of hard X rays of energies (20-200 keV)

As it is shown in the book of Heitler (The Quantum Theory of Radiation, Clarendon, Oxford, 1960), it is more convenient to made the analysis in the inertial system in which the initial velocity of the electrons is zero.

There are two geometries of interaction, the first, when angle between the wave vector and the electron velocity is 180^0, and the second, when this angle is 90^0.

$S(t,x,y,z)$

$S'(t',x',y',z')$

S

x

$\bar{E}_L$

$\bar{k}_1$ o $\bar{k}_L$ z

$\bar{B}_1$

y $\bar{B}_L$ $\bar{E}_1$

S'

x'

$\bar{E}'_L$

$\overline{V}_0$ $\bar{k}'_1$ o $\bar{k}'_L$ z'

$\bar{B}'_1$

y' $\bar{B}'_L$ $\bar{E}'_1$

180^0 geometry

S

x

$\bar{E}_L$

$\bar{k}_1$

$\bar{E}_1$ o $\bar{k}_L$ z

$\bar{B}_1$

y $\bar{B}_L$

S'

x'

$\overline{V}_0$

$\bar{k}'_1$

$\bar{E}'_1$ o $\bar{E}'_L$ z'

$\bar{B}'_1$

y' $\bar{B}'_L$ $\bar{k}'_L$
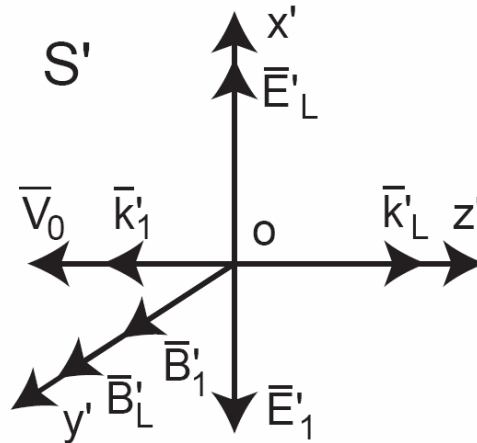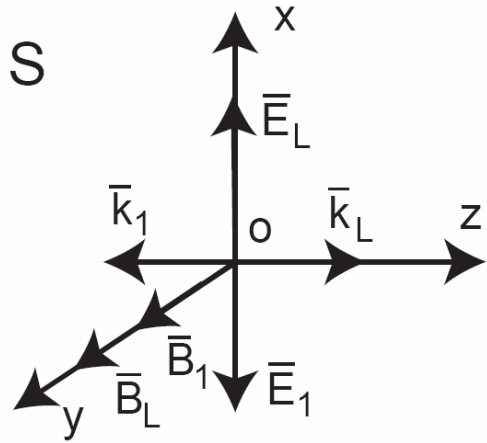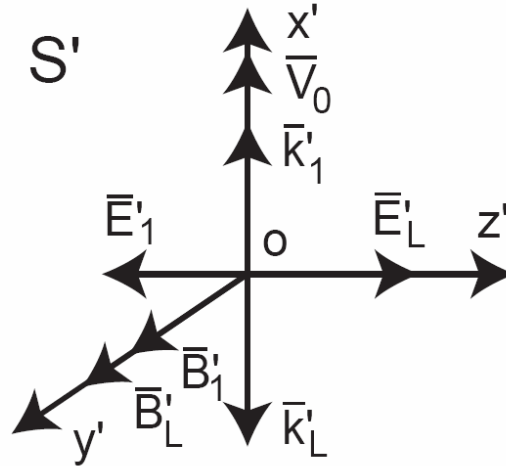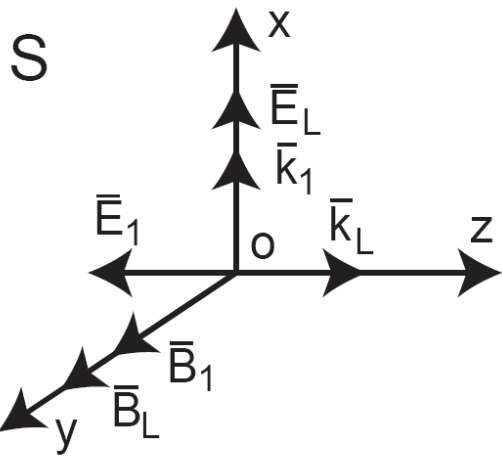
90^0 geometry

First stage: calculation of the laser field parameters in the system
S.' We limit at the case of the 180^0 geometry

$$S(t, x, y, z) \qquad S'(t', x', y', z') \qquad \bar{\beta}_0 = \frac{\bar{V}_0}{c} = -|\beta_0|\bar{k} \qquad \gamma_0 = \frac{1}{\sqrt{1 - \frac{V_0^2}{c^2}}}$$

$$\left(\frac{\omega_L}{c}, k_{Lx}, k_{Ly}, k_{Lz}\right) \qquad \left(\frac{\omega_L'}{c}, k_{Lx}', k_{Ly}', k_{Lz}'\right)$$

ω_L

$k_{Lx} = k_{Ly} = 0$

$k_{Lz} = |k_L|$

$E_{Lx} = E_L$

$E_{Ly} = E_{Lz} = 0$

$B_{Ly} = B_L = \dfrac{E_L}{c}$

$B_{Lx} = B_{Lz} = 0$

$\omega_L' = \omega_L \gamma_0 (1 + |\beta_0|)$

$k_{Lx}' = k_{Ly}' = 0$

$k_{Lz}' = |k_L| \gamma_0 (1 + |\beta_0|)$
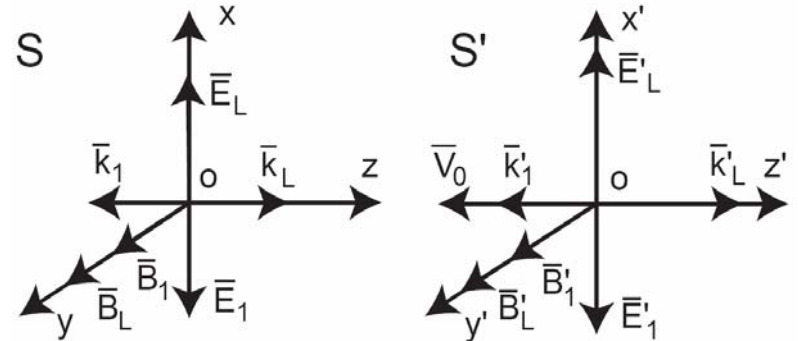
$E_{Lx}' = E_L \gamma_0 (1 + |\beta_0|) = E_L'$

$E_{Ly}' = E_{Lz}' = 0$

$B_{Ly}' = B_L \gamma_0 (1 + |\beta_0|) = B_L'$

$B_{Lx}' = B_{Lz}' = 0$

$$a' = \frac{eE_M'}{mc\omega'} = \frac{eE_M}{mc\omega} = a \qquad \eta' = \omega't' - |k'|z' = \omega_1 t - |k|z = \eta$$

Second stage: solution of the equations of the electron motion in the system S'

$$\frac{d}{dt'}(\gamma'\beta_x') = -a'\omega'(1-\beta_z')\cos\eta'$$

$$mc\frac{d}{dt'}(\gamma'\beta_z') = -ec\beta_x'B_L'$$

$$\gamma' = \left(1 - \beta_x'^2 - \beta_z'^2\right)^{-\frac{1}{2}}$$

$$t' = 0 \qquad x' = y' = z' = 0 \qquad \beta_x' = \beta_y' = \beta_z' = 0$$

$$E_L' = E_M'\cos\eta' \qquad B_L' = B_M'\cos\eta'$$

$$E_M' = E_M\gamma_0(1+|\beta_0|) \qquad B_M' = B_M\gamma_0(1+|\beta_0|)$$
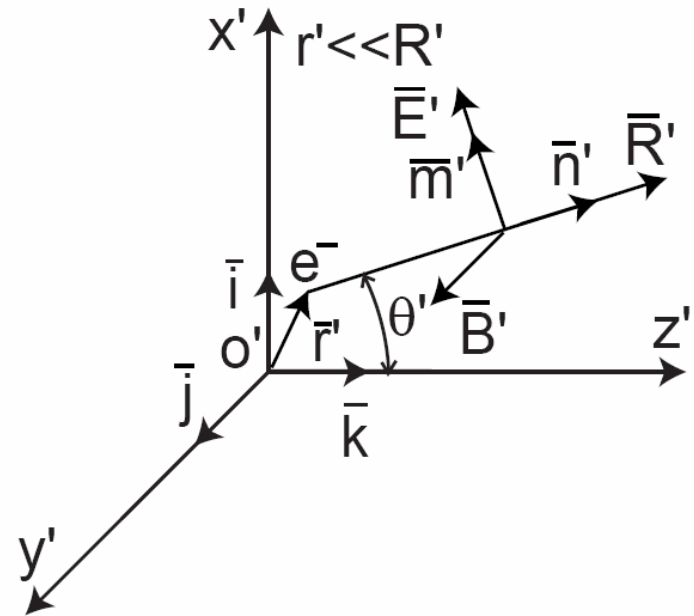
Remarkable property of the solution: it is function only of η'

Result: β_x' β_z' $\dot{\beta}_x'$ $\dot{\beta}_z'$ as periodic functions of η'

<u>Third stage</u>: calculation of the parameters of the field generated in the S' system

$$\overline{E}' = \frac{-e}{4\pi\varepsilon_0 cR'} \cdot \frac{\overline{n}' \times \left[\left(\overline{n}' - \overline{\beta}'\right) \times \dot{\overline{\beta}}'\right]}{\left(1 - \overline{n}' \cdot \overline{\beta}'\right)^3} \qquad\qquad \overline{E}' = -E_M{}' \frac{r_0}{R'} f(\eta', \theta')\overline{m}'$$

$$f(\eta', \theta') = \frac{\cos\theta + \dfrac{a'^2 \sin^2\eta'}{2}(1 - \cos\theta') + a'\sin\eta'\sin\theta'}{\left(1 + \dfrac{a'^2 \sin^2\eta'}{2}(1 - \cos\theta') + a'\sin\eta'\sin\theta'\right)^3}\cos\eta'$$

$$\overline{E}' = \sum_{j=1}^{\infty}\overline{E}_j{}' \qquad \overline{B}' = \sum_{j=1}^{\infty}\overline{B}_j{}' \qquad \omega_j{}' = j\omega_L{}' = j\gamma_0\left(1 + |\beta_0|\right)\omega_L \qquad \overline{k}_j{}' = -j\gamma_0\left(1 + |\beta_0|\right)k_L\overline{k}$$

<u>T</u>he experimental data from literature correspond to a'=a<1. In this case the first harmonic is dominant. We limit the analysis at the first harmonic, when θ=π, in the case of the 180⁰ geometry.

$$\omega_1{}' = \gamma_0\left(1 + |\beta_0|\right)\omega_L \qquad \overline{k}_1{}' = -\gamma_0\left(1 + |\beta_0|\right)k_L\overline{k}$$

$$\overline{E}_1{}' = -E_1{}'\overline{i} \qquad \overline{B}_1{}' = B_1{}'\overline{j} = \frac{E_1{}'}{c}\overline{j} \qquad E_1{}' = -E_M{}'\frac{r_0}{R'}f_1\cos\eta' \qquad f_1 = \frac{1}{\pi}\int_0^{2\pi}\frac{-1 + a'^2\sin^2\eta'}{\left(1 + a'^2\sin^2\eta'\right)^3}\cos^2\eta'\cdot d\eta'$$

The angle of divergence of X-rays beam in the S' system: Δθ'=π/4

Fourth stage: calculation of the parameters of the field generated in the laboratory system

$$S'(t', x', y', z')$$

$$\left(\frac{\omega_1'}{c}, 0, 0, -|k_1'|\right)$$

$$\omega_1' = \gamma_0\left(1 + |\beta_0|\right)\omega_L$$

$$k_{1z}' = k_1' = -|k_L|\gamma_0\left(1 + |\beta_0|\right)$$

$$k_{1x}' = k_{1y}' = 0$$

$$E_{1x}' = -E_1'$$

$$E_{1y}' = E_{1z}' = 0$$

$$B_{1y}' = B_1'$$

$$B_{1x}' = B_{1z}' = 0$$

$$S(t, x, y, z)$$

$$\left(\frac{\omega_1}{c}, k_{1x}, k_{1y}, k_{1z}\right)$$

$$\omega_1 = \omega_L\gamma_0^2\left(1 + |\beta_0|\right)^2$$

$$k_{1z} = k_1 = -|k_L|\gamma_0^2\left(1 + |\beta_0|\right)^2$$
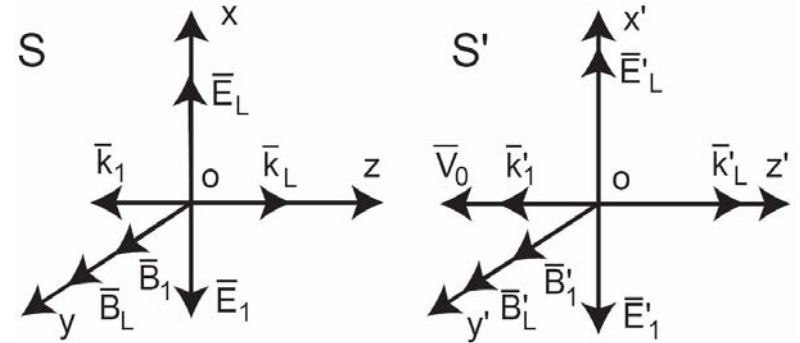
$$k_{1x} = k_{1y} = 0$$

$$E_{1x} = E_1 = -\gamma_0 E_1'\left(1 + |\beta_0|\right)$$

$$E_{1y} = E_{1z} = 0$$

$$B_{1y} = B_1 = \gamma_0 \frac{E_1'}{c}\left(1 + |\beta_0|\right)$$

$$B_{1x} = B_{1y} = 0$$

$$\Delta\theta = \frac{1}{\gamma_0}$$

$$a_1' = \frac{eE_{M1}'}{mc\omega_1'} = \frac{eE_{M1}}{mc\omega_1} = a_1$$

$$\eta_1' = \omega_1't' + |k_1'|z' = \omega_1 t + |k_1|z = \eta_1$$

Final relations:

180⁰ geometry:

$$W_{X1} = \omega_1 \hbar = \omega_L \gamma_0^2 \left(1 + |\beta_0|\right)^2 \hbar$$

$$\lambda_{X1} = \frac{\lambda_L}{\gamma_0^2 \left(1 + |\beta_0|\right)^2}$$

90⁰ geometry:

$$W_{X1} = \omega_1 \hbar = \omega_L \gamma_0^2 \left(1 + |\beta_0|\right) \hbar$$

$$\lambda_{X1} = \frac{\lambda_L}{\gamma_0^2 \left(1 + |\beta_0|\right)}$$

Input data: W_e, W_L, λ_L, r_L, τ_L

$$\gamma_0 = \frac{W_e}{mc^2} \qquad \beta_0 = \sqrt{1 - \frac{1}{\gamma_0^2}} \qquad I_L = \frac{W_L}{\tau_L \pi \cdot r_L^2} \qquad E_M = \sqrt{\frac{2 I_L}{\varepsilon_0 c}}$$

$$\omega_L = \frac{2\pi \cdot c}{\lambda_L} \qquad a = \frac{e E_M}{mc\omega} \qquad \Delta\theta = \frac{1}{\gamma_0}$$

Table 1. Comparison between theoretical and experimental values of the X-rays photon energies and wavelengths, denoted, respectively, by W_{X1}, λ_{X1} and W_{X1exp}, λ_{X1exp}.

Characteristics	Case 1	Case 2	Case 3	Case 4
Geometry	180^0	180^0	180^0	90^0
$W_e [MeV]$	60	55	57	32
γ_0	117.4	107.6	111.5	62.62
$|\beta_0|$	0.99996	0.99996	0.99996	0.99987
$W_L [J]$	0.2	0.180	0.180	0.2
$\lambda_L [\mu m]$	10.64	0.800	0.800	0.800
$r_L [\mu m]$	32	35	32	50
$\tau_L [s]$	180×10^{-12}	54×10^{-15}	54×10^{-15}	170×10^{-15}
$I_L [W/m^2]$	3.454×10^{17}	8.661×10^{20}	8.661×10^{20}	1.498×10^{20}
$E_M [V/m]$	1.613×10^{10}	8.078×10^{11}	8.078×10^{11}	3.3596×10^{11}
$\omega_L [s^{-1}]$	1.770×10^{14}	2.355×10^{15}	2.355×10^{15}	2.355×10^{15}
a	0.05346	0.2012	0.2012	0.08371
$W_{X1} [keV]$		71.8	77.1	
$W_{X1exp} [keV]$		73.1	78.5	
$\lambda_{X1} [\mathring{A}]$	1.9			1.02
$\lambda_{X1exp} [\mathring{A}]$	1.8			1
Reference	[1] pag. 090702-7	[2] pag. 894	[2] pag. 894	[3] pag. 354

[1]. Phys. Rev. ST-AB, 3, 090702, 2000 (Brookhaven N. L.). [2]. Appl. Phys. B, 78, 891, 2004 (Lawrence Livermoore N.L.) [3]. Nucl. Instr. Meth. In Phys. Res. A, 341 351, 1994 (Lawrence Berkeley Lab.)

FIRST RESULT – GRANT CNCSIS

We consider the Schrodinger equation written for a closed system composed by electromagnetic linear polarized field and an electron, when the interaction term and the energy of the e. m. field enter in their classical form

$$\left[\frac{1}{2m} \left(-i\hbar\nabla + e\overline{A} \right)^2 + E_{em} \right] \Psi_0 = E\Psi_0 \qquad \text{where} \qquad E_{em} = \frac{1}{2} \int_V \left[\varepsilon_0 \left(\frac{\partial\overline{A}}{\partial t} \right)^2 + \frac{1}{\mu_0} \left(\nabla \times \overline{A} \right)^2 \right] dV$$

We consider also the Hamilton-Jacobi equation written for the same system

$$\frac{1}{2m} \left(\nabla S_0 + e\overline{A} \right)^2 + E_{em} = E$$

We proved the following property:

If the dipol approximation is fulfilled and the Hamilton-Jacobi equation has the solution $\sigma = S_0$, then the Schrodinger equation is verified by the function

$$\Psi_0 = \exp\left(\frac{i\sigma}{\hbar} \right)$$

This property is published in IEEE J. Quantum Electronics, 43, 1183, 2007

This property is proved without using the semiclassical (WKB) approximation or the approximation of the geometrical optics

CONCLUSION There is a direct connection between the quantum and classical functions, Ψ_0 and S_0 which correspond to the same value of the total energy. **A similar connection is valid in the case of stationary systems, as follows.**

SECOND RESULT.

We consider a stationary bound system whose behavior is described by the Schrödinger equation. The total energy E is constant and negative and the potential energy U does not depend explicitly on time.

For this system the Schrödinger equation is rigorously equivalent to the wave equation.

We proved the following properties:

1) The characteristic surfaces of the wave equation, denoted by Σ, and their normals, denoted by C, are solutions of the Hamilton-Jacobi equation written for the same system.

2) The constants of motion corresponding to C curves, including the total energy E, are identical to the eigenvalues of the Schrodinger equation.

<u>It results again direct connection between quantum and classical functions, Ψ_0 and S_0 (corresponding to the C curve), which correspond to the same value of the total energy. The similitude between the two results is almost perfect.</u>

The idea of our approach is to use the properties of the C curves, to calculate the properties of the system.

Calculations based on the properties of the C curves are presented in the following papers:

[1]. A. Popa, Wave model for conservative bound systems, <u>Journal of Chemical Physics</u>, 122, 244701 (2005).

[2]. A. Popa, Connection between the periodic solutions of the Hamilton-Jacobi equation and the wave properties of the conservative bound systems, <u>Journal of Physics A: Mathematical and General</u>, 36, 7569-7578 (2003).

[3]. A. Popa, Applications of a Property of the Schrödinger Equation to the Modeling of Conservative Discrete Systems, <u>Journal of the Physical Society of Japan</u>, 67, 2645-2652 (1998).

[4]. A. Popa, Applications of a Property of the Schrödinger Equation to the Modeling of Conservative Discrete Systems. II, <u>Journal of the Physical Society of Japan</u>, 68, 763-770 (1999).

[5]. A. Popa, Applications of a Property of the Schrödinger Equation to the Modeling of Conservative Discrete Systems. III, <u>Journal of the Physical Society of Japan</u>, 68, 2923-2933 (1999).

[6]. A. Popa, A Remarkable Property of the Schrodinger Equation, <u>Revue Roumaine de Mathematiques Pures et Appliquees</u>, 41, 109-117 (1996).

[7]. A. Popa, A Property of the Hamilton-Jacobi Equation, <u>Revue Roumaine de Mathematiques Pures et Appliquees</u>, 43, 415-424 (1998).

[8]. A. Popa, Intrinsic Periodicity of Conservative Systems, <u>Revue Roumaine de Mathematiques Pures et Appliquees</u>, 44, 119-122 (1999).

[9]. A. Popa, An Accurate Wave Model for Conservative Molecular Systems, in Multiphoton and Light Driven Multielectron Processes in Organics: Materials Phenomena, Applications, eds. F. Kajzar and M.V. Agranovich, Kluwer Academic, Dordrecht, 2000, p. 513-526.

[10]. A. Popa, Semiclassical method for calculating the energetic values of helium, lithium and beryllium atoms, <u>European Physical Journal D</u>, 49, 2008, in print.

Table 1. Normalized values of the total energy calculated in this paper and in literature, with the aid of Hartree-Fock methods, as compared to experimental values. The values are given in Rydbergs. The corresponing values of s_{31e} and s_{13e} are given in Table 2.

State	E (this paper)	E (literature)	E_{exp}
He $1s^2$	-5.83562	-5.7233402 [20]	-5.806921 [15]
		-5.723359 [22]	
		-5.7233598 [23]	
		-5.7233600 [25]	
		-5.7233600 [27]	
Li $1s^2 2s$	-14.9559	-14.8654514 [23]	-14.956336 [15]
		-14.8654538 [25]	
		-14.8654516 [27]	
		-14.8654538 [26]	
		-14.8654475 [28]	
Li $1s^2 2p$	-14.8172	-14.730136 [19]	-14.82147 [16]
		-14.760174 [21]	
		-14.760382 [24]	
Be $1s^2 2s^2$	-29.2517	-29.144736 [20]	-29.337659 [15]
		-29.146044 [22]	
		-29.146042 [23]	
		-29.146042 [27]	
		-29.14603376 [28]	

In the Chapter 4, Section 7, entitled "The critic of our preliminary interpretation of the quantum theory in terms of hidden variables", of his last book "Wholeness and the Implicate Order," David Bohm showed that it is necessary to find a solution where the quantum potential does not appear.

The author has introduced in a series of papers a very interesting way of relating quantum mechanics to classical mechanics avoiding the pitfalls of Bohm-type potentials. In the present paper submitted for review, the author presents a number of calculations for atoms and compares the energy obtained semiclassically with those obtained by strict quantum mechanical methods.