Directii de cercetare in mecanica solidelor: traditii si perspective

I: Modelare in plasticitatea finita

Sanda Cleja-Tigoiu
Universitea Bucuresti

Workshop: Directii actuale si de perspectiva in cercetarea matematica

Directii:

Modelarea matematica in mecanica mediilor solidelor are premizele in natura fizica a fenomenelor si in bazele experimentale.

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Metodele numerice elaborate de mecanica computationala .

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- A mathematical formalism alone cannot lead to physically realistic theories. It has to be accompanied by physical investigations, and requires good insight into the physical situation, in particular on the microscale.
- Differential geometry is a most important and elegant tool to deal with the internal mechanical state, both on the geometric and static side.

Directii in plasticitatea finita:

Plasticitatea s-a dezvolat in formalismul micilor deformatii , pentru materiale de tip rate si materiale rigid-vascoplastice (sau bazate pe legi constitutive de tip fluid) (pana prin anii 80),

S. C-T., N. Cristescu, Plasticitatate cu aplicatii..Ed. Universitatii, (1985)

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Plasticitate in formalismul deformatiilor finite :

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Au fost formalizate si analizate probleme legate de

anizotropia initiala si anizotropia indusa de dezvoltarea si acumularea deformatiilor plastice,

natura disipativa a deformarii plastice,

rolul spinului plastic;

bifurcarea solutiilor in plasticitatea cristalelor,

formularea inegalitatilor variationale asociate problemelor cuasi-statice cu date date la limita, intr-un stadiu generic al procesului de deformare, etc.

Directii in plasticitatea finita:

II. Mecanica configurationala (sau materiala) s-a dezvoltat prin

incercarile de descriere la nivel macroscopic a schimbarilor structurale produse de existenta neomogenitatilor microstructurale, si prin

tendintele de unificare a studiului anumitor fenomene, ca deteriorare si plasticitate, plasticitate si dislocatii,

twining si plasticitate (colaborare cu Prof. K. Rajagopal, Program Fulbright) a fost concretizat in S.C-T, ZAMP(submitted) si continuat S.C-T, Int.J. Fracture, (2007)

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A fost dezvoltat un formalism matematic, prin teoriile de plasticitate de ordinul al doilea,

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A fost dezvoltat un formalism matematic, prin teoriile de plasticitate de ordinul al doilea,

bazat pe configuratii locale, neolonome, care iau in consideratie neomogenitatile structurale, de tipul

dislocatii continuu distribuite , sau

prezenta unor zone de deteriorare .

II. Cum descriem comportamentul materialului e-p.?

Bazat pe existenta configuratiilor locale cu torsiune

Corpul $\mathcal{B}$ este considerat o varietate diferentiabila, conexa, dim $\mathcal{B}=$ n.

Difeomorfismele ϕ definite pe $\mathcal{B}$ se numesc deformatii, $\phi(\mathcal{B})$ configuratii,

$\{\chi(\cdot,t)\}_{t\in R}\}$ set de difeomorfisme, defineste o miscare a lui $\mathcal{B}$

$k(\mathcal{B})$ o configuratie de referinta fixata.

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H1.$(\exists)$ Deformatie (plastica) de ordinul doi

$$\forall \quad \chi \quad \text{miscare} \quad \text{a corpului} \quad \mathcal{B} \quad \forall \mathbf{X}, \quad \forall t \quad \exists \quad (\mathbf{F}^p, \mathbf{\Gamma}^p)$$

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$\mathbf{F}^p : \mathcal{T}_{\mathbf{X}} \to \mathcal{V}_{\mathcal{K}}$ liniara, inversibila, numita distorsiune plastica

$\mathbf{\Gamma}^p : \mathcal{T}_{\mathbf{X}} \longrightarrow Lin(\mathcal{T}_{\mathbf{X}}, \mathcal{T}_{\mathbf{X}})$ liniara, numita conexiunea plastica, cu torsiune nenula i.e.

$$(\mathbf{S}_k\mathbf{u})\mathbf{v} = (\mathbf{\Gamma}_k\mathbf{u})\mathbf{v} - (\mathbf{\Gamma}_k\mathbf{v})\mathbf{u} \quad \forall \mathbf{u}, \mathbf{v}. \qquad (1)$$

exista $\mathcal{K}_t \equiv \mathcal{K}$ configuratie cu torsiune $\quad \Longleftrightarrow$

$(\mathbf{F}^p-$ distorsiunea plastica, $\overset{(p)}{\mathbf{\Gamma}}_k -$ conexiunea cu torsiune).

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- Ecuatii de evolutie pentru definirea $\mathcal{K}_t-$ configuratiei cu torsiune si a variabilelor interne de stare.

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- Ecuatii de evolutie pentru definirea $\mathcal{K}_t-$ configuratiei cu torsiune si a variabilelor interne de stare.
- Principii energetice:

  principiului puterii virtuale $\Longrightarrow$ Ecuatii macro si micro de bilant.

  principii de disipare sau de nebilantare energetica $\Longrightarrow$ restrictii termomecanice

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- Models with Non-Riemannian connections:

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- S. C-T., in *New Trend in Contiuum Mech.*, Ed. M. Mihailescu-Suliciu (2005)

[2] Compunerea deformatiilor de ordinul doi

$\exists$ deformatia elastica de ordinul doi astfel incat

$$(\mathbf{F}, \mathbf{\Gamma}) := (\mathbf{F}^e, \overset{(e)}{\mathbf{\Gamma}}_{\mathcal{K}}) \circ (\mathbf{F}^p, \overset{(p)}{\mathbf{\Gamma}}_k), \quad \Longleftrightarrow$$

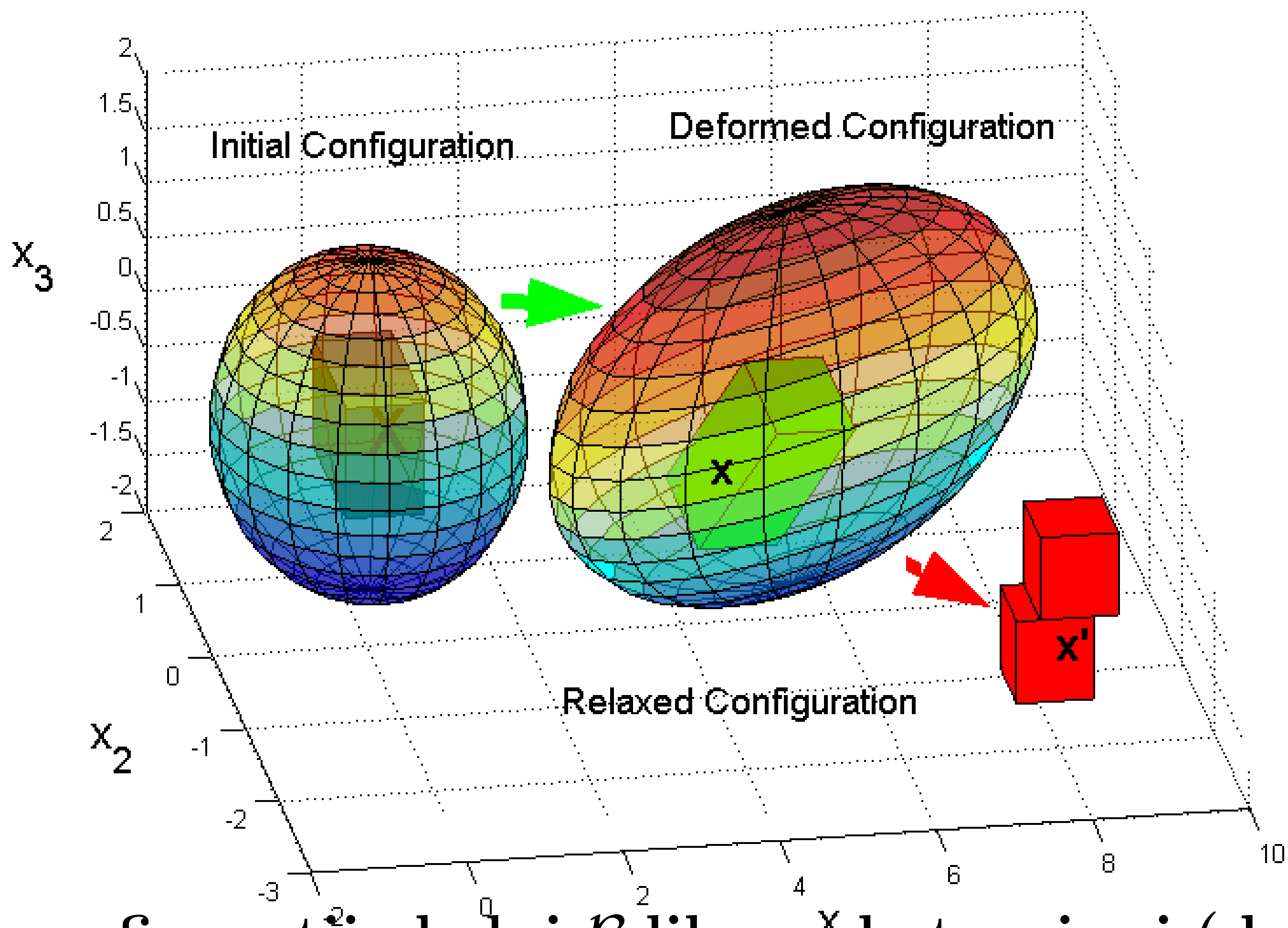
$$\mathbf{F} = \mathbf{F}^e \mathbf{F}^p \quad \text{descompunerea multiplicativa} \tag{2}$$

$$\mathbf{\Gamma} = \mathbf{F}^p \overset{(e)}{\mathbf{\Gamma}}_{\mathcal{K}} [(\mathbf{F}^p)^{-1}, (\mathbf{F}^p)^{-1}] + \overset{(p)}{\mathbf{\Gamma}}_k, \quad \text{compunerea conexiunilor},$$

Notatie: $((\mathbf{\Gamma}[\mathbf{F}^p, \mathbf{F}^p])\mathbf{u})\mathbf{v} = (\mathbf{\Gamma}(\mathbf{F}^p\mathbf{u}))\mathbf{F}^p\mathbf{v}$.

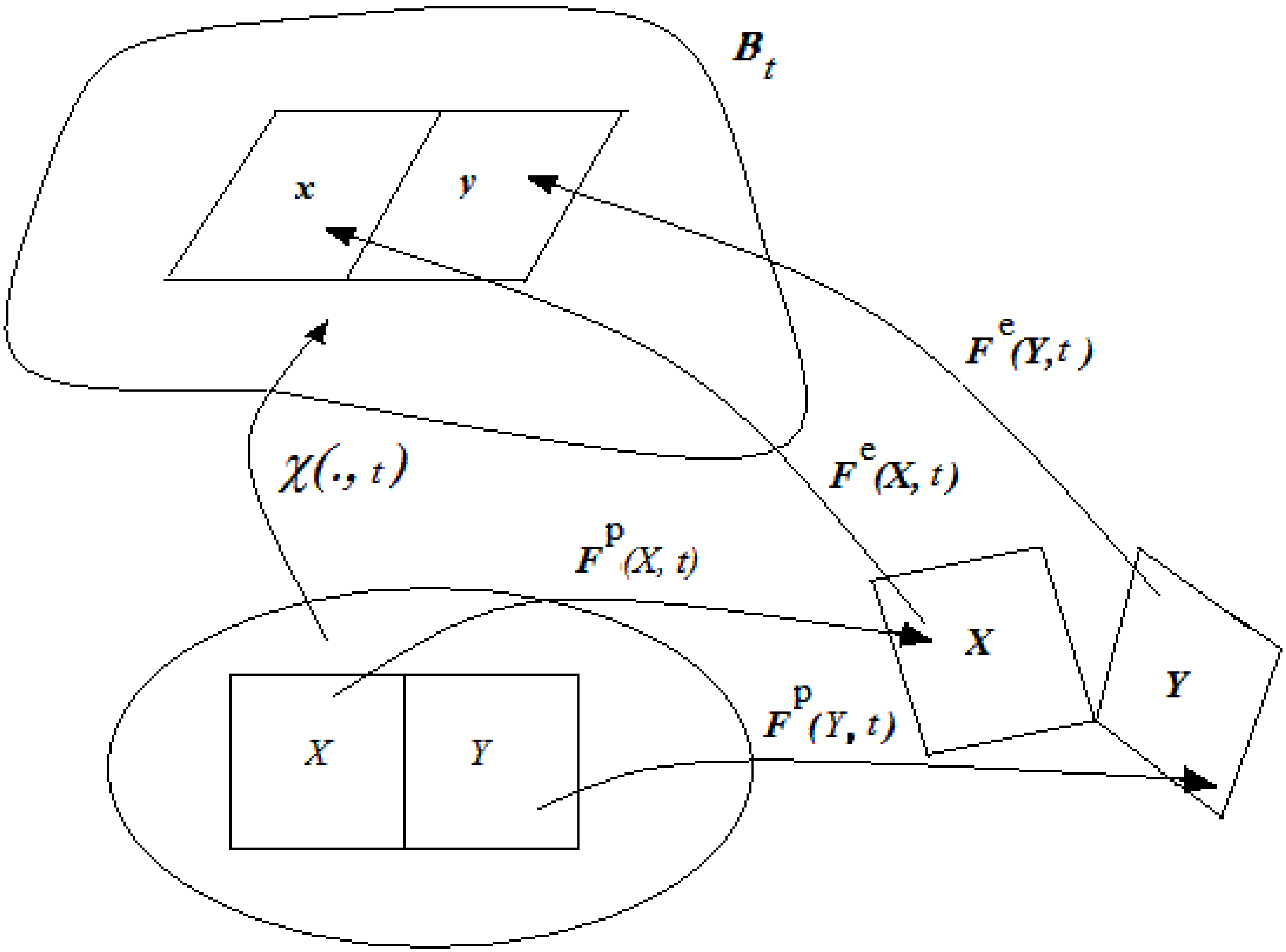
$(\mathbf{F}, \mathbf{\Gamma}) := (\nabla\chi(\mathbf{X}, t), \quad \mathbf{F}^{-1}(\nabla\mathbf{F})$ asociata miscarii $\chi(\cdot, t)$.

2.1 Configuratii locale libere de tensiuni

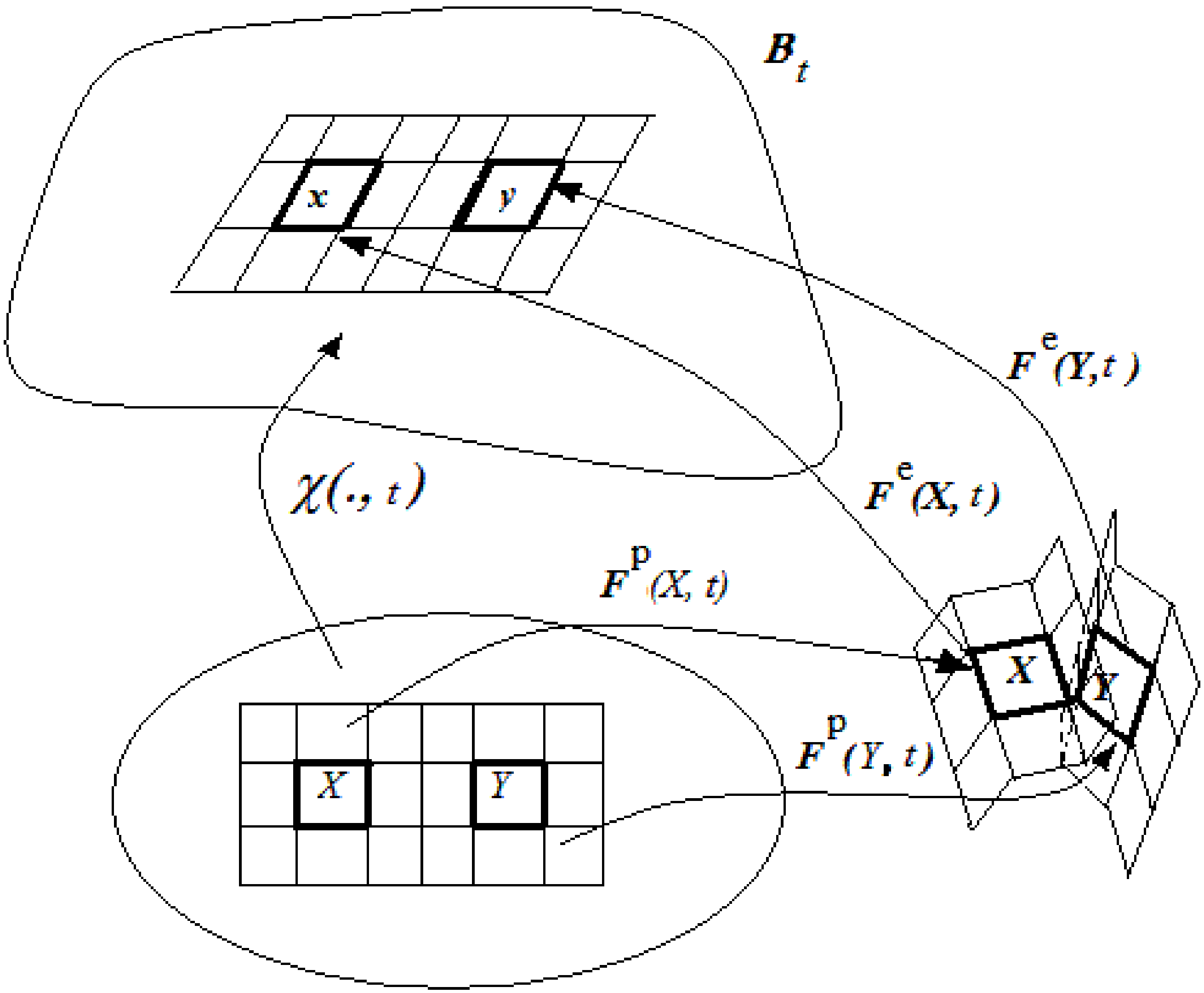


Nu exista configuratii ale lui $\mathcal{B}$ libere de tensiuni (deformate plastic).

2.1 Configuratii (locale) cu torsiune si fara curbura



2.1 Configuratii (locale) cu torsiune si curbura nenule



2.2 Prezenta dislocatiilor

modelata prin conditia de ne-integrabilitate

i.e. (1) cu rotor nenul

$$(1)\ \text{curl}(\mathbf{F}^p) \neq 0 \quad \text{or}$$

$$(\nabla \mathbf{F}^p(\mathbf{u}))\mathbf{v} - (\nabla \mathbf{F}^p(\mathbf{v}))\mathbf{u} \neq 0 \quad \exists \mathbf{u}, \mathbf{v},$$

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sau (2) torsiune plastica nenula

$$(2)\ (\mathbf{S}\mathbf{u})\mathbf{v} = (\mathbf{\Gamma}^p_k \mathbf{u})\mathbf{v} - (\mathbf{\Gamma}^p_k \mathbf{v})\mathbf{u} \neq 0 \quad \exists \mathbf{u}, \mathbf{v}. \tag{3}$$

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Conexiunea plastica compatibila cu distorsiunea plastica daca campul tensorial $\mathbf{F}^p$ este solutie a PD-Eqn.

$$\overset{(\mathrm{p})}{\mathbf{\Gamma}}_k = (\mathbf{F}^p)^{-1} \nabla_k \mathbf{F}^p, \quad \forall \quad \mathbf{X} \in \mathcal{U}.$$

- Conditia de integrabilitate de tip Frobenius daca tensorul de curbura Riemann $\mathcal{R} = 0$.

Teorema asupra torsiunii

Bilby (1960) considera cazul curburii zero, pentru conexiuni care descriu metricitatea deformatiilor si dislocatiile.

PD-Eqn. pentru torsiune compatibila cu curbura Riemann zero dedusa in S. C-T, D. Fortune, C. Valle, *M.M.S.* (2008)

Teorema asupra torsiunii

Conexiunea $\mathbf{\Gamma}$ cu torsiune nenula este caracterizata de $\mathbf{C}-$ tensor metric si $\mathbf{\Lambda}-$ tensor de ordinul doi prin

$$\mathbf{\Gamma u} = \mathbf{U}^{-1}(\mathbf{\Lambda u} \times \mathbf{I})\mathbf{U} + \mathbf{U}^{-1}(\nabla \mathbf{U})\mathbf{u}. \tag{4}$$

cu $\mathbf{U}$ astfel incat $\mathbf{U}^2 = \mathbf{C}$.

[•] $\exists$ $\mathbf{R}-$ ortogonal astfel incat $\mathbf{\Gamma}$ este compatibl cu $\mathbf{F} := \mathbf{RU}$, daca si numai daca $\mathbf{\Lambda}$ este solutie pentru PD-Eqn.

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- In cazul micilor deformatii plastice conditia de integrabilitate conduce la PD-Eqn. pentru $\mathbf{N}$

$$-\text{curl } \mathbf{N} + \text{curl } (\text{curl } \boldsymbol{\epsilon}^p) = 0, \tag{6}$$

$\boldsymbol{\epsilon}^p-$ tensorul simetric al micilor deformatii.

- $\mathbf{N} = 0$ datorat lui Krönner.

Vector Burgers $\mathbf{b}_{\mathcal{K}}$

- exprimat prin distorsiunea plastica $\mathbf{F}^p$

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$$
\begin{aligned}
\mathbf{b}_{\mathcal{K}} &:= \int_{C_0} \mathbf{F}^p \, d\mathbf{X} = \\
&= \int_{\mathcal{A}_0} \text{curl}(\mathbf{F}^p)\mathbf{N} dA = \int_{\mathcal{A}_{\mathcal{K}}} \boldsymbol{\alpha}_{\mathcal{K}} \mathbf{n}_{\mathcal{K}} dA_{\mathcal{K}},
\end{aligned} \tag{7}
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$$\boldsymbol{\alpha}_{\mathcal{K}} \equiv \frac{1}{\det \mathbf{F}^p} \text{curl}(\mathbf{F}^p)(\mathbf{F}^p)^T \quad \text{dislocatii in sensul lui Noll} \tag{8}$$

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- Prezenta dislocatiilor (defectelor in structura cristalina) este carcterizata prin vector Burgers $\mathbf{b}_{\mathcal{K}} \neq 0$.

[1] References

- Incompatibilities:

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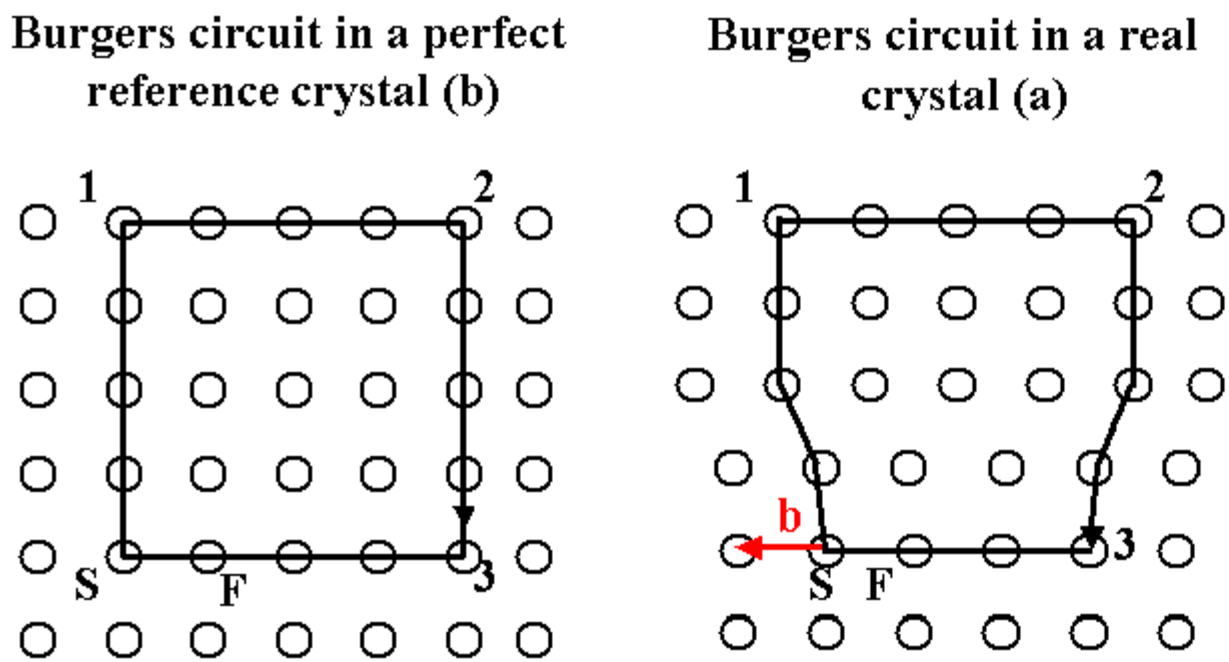
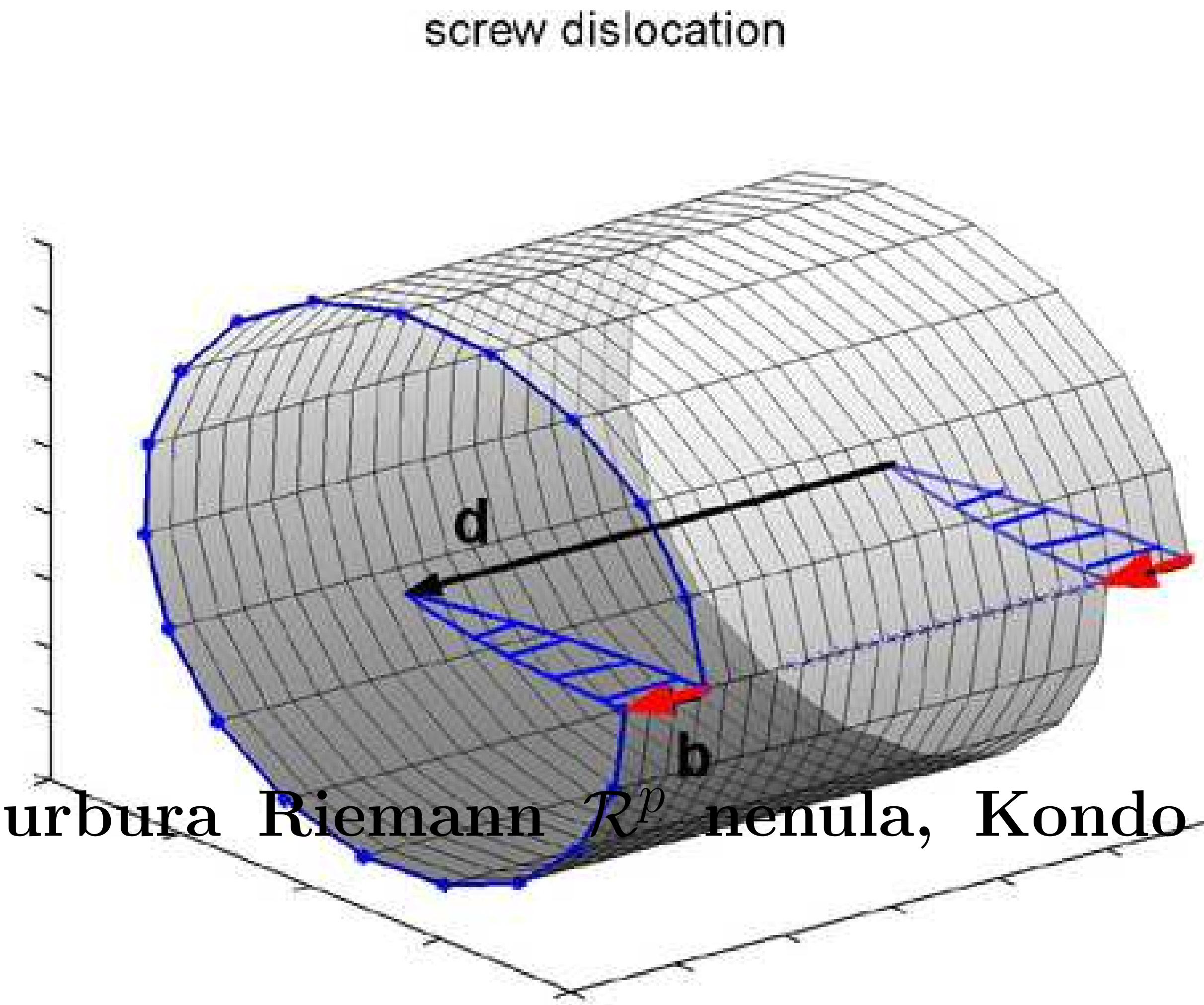
Dislocatii unghiulare, vector Burgers



Dislocatii: torsiunea $\mathbf{S}_{\mathcal{K}}$ nenula.

Dislocatie de tip surub, vector Burgers



Disclinatie: curbura Riemann $\mathcal{R}^p$ nenula, Kondo si Yuki (1958).

[3] Conexiuni cu proprietate de metricitate si de nemetricitate

In configuration cu torsiune $\mathcal{K}$, doua structuri geometrice pot fi introduse

$$((\mathbf{F}^p)^{-1}, \overset{(\mathrm{p})}{\mathbf{\Gamma}}_{\mathcal{K}}), \quad \text{cu tensorul metric} \quad \mathbf{c}^p := (\mathbf{F}^p)^{-T}(\mathbf{F}^p)^{-1}$$

$$(\mathbf{F}^e, \overset{(\mathrm{e})}{\mathbf{\Gamma}}_{\mathcal{K}}), \quad \text{cu tensorul metric} \quad \mathbf{C}^e := (\mathbf{F}^e)^T(\mathbf{F}^e). \tag{9}$$

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Conexiunea $\overset{(\mathrm{p})}{\mathbf{\Gamma}}_{\mathcal{K}}$ are proprietate de metricitate in raport cu tensorul metric $\mathbf{c}^p$ (Schouten (1954))

$$(\nabla_{\mathcal{K}}\, \mathbf{c}^p)\mathbf{u} = (\overset{(\mathrm{p})}{\mathbf{\Gamma}}_{\mathcal{K}}\, \mathbf{u})^T \mathbf{c}^p + \mathbf{c}^p(\overset{(\mathrm{p})}{\mathbf{\Gamma}}_{\mathcal{K}}\, \mathbf{u}), \quad \forall \quad \mathbf{u}, \tag{10}$$

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si respectiv de nemetricitate, daca exista un camp tensorial de ordinul trei $\mathbf{Q}$ astfel incat

$$\mathbf{Q}\mathbf{u} \equiv (\overset{(\mathrm{p})}{\mathbf{\Gamma}}_{\mathcal{K}}\, \mathbf{u})^T \mathbf{c}^p + \mathbf{c}^p(\overset{(\mathrm{p})}{\mathbf{\Gamma}}_{\mathcal{K}}\, \mathbf{u}) - (\nabla_{\mathcal{K}}\, \mathbf{c}^p)\mathbf{u}. \tag{11}$$

Teoreme de descompunere a conexiunilor

Axioma. Conexiunea plastica in $\mathcal{K}$ are proprietati metrice in raport tensorul (plastic) metric $\mathbf{c}^p$.

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$$\overset{(\mathrm{p})}{\mathbf{\Gamma}}_{\mathcal{K}} = \boldsymbol{\gamma}^p_{\mathcal{K}} + \mathbf{W}_{\mathcal{K}}, \quad \mathbf{W}_{\mathcal{K}} = (\mathbf{c}^p)^{-1}\overline{\mathbf{W}}_{\mathcal{K}}. \qquad (12)$$

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- cu anti-simetriile:

$$\overline{\mathbf{W}}_{\mathcal{K}}(\mathbf{u}) = -(\overline{\mathbf{W}}_{\mathcal{K}}(\mathbf{u}))^T, \quad (\mathbf{S}_{\mathcal{K}}\mathbf{u})\mathbf{v} = -(\mathbf{S}_{\mathcal{K}}\mathbf{v})\mathbf{u} \quad \forall \mathbf{u}, \mathbf{v}.$$

Teoreme de descompunere a conexiunilor

Teorema 2. In urmatoarele ipoteze:

a. conexiunea plastica are proprietati metrice,

b. compunerea deformatiilor elastice si plastice este definita prin formula ().

(1) Conexiunile plastice si repectiv elastice pot fi reprezentate prin

$$\overset{(p)}{\mathbf{\Gamma}}_{\mathcal{K}}\,\tilde{\mathbf{u}} = \underbrace{\mathbf{F}^p(\nabla_{\mathcal{K}}(\mathbf{F}^p)^{-1})}\,\tilde{\mathbf{u}} + (\mathbf{c}^p)^{-1}(\mathbf{\Lambda}\tilde{\mathbf{u}} \times \mathbf{I})$$

$$\overset{(e)}{\mathbf{\Gamma}}_{\mathcal{K}}\,\tilde{\mathbf{u}} = \underbrace{(\mathbf{F}^e)^{-1}(\nabla_{\mathcal{K}}(\mathbf{F}^e))}\,\tilde{\mathbf{u}} + (\mathbf{c}^p)^{-1}(\mathbf{\Lambda}\tilde{\mathbf{u}} \times \mathbf{I}). \tag{14}$$

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Introducem urmatoarele notatii

$$\overset{(p)}{\boldsymbol{\mathcal{A}}}_{\mathcal{K}}\,\tilde{\mathbf{u}} := \mathbf{F}^p(\nabla_{\mathcal{K}}(\mathbf{F}^p)^{-1})\tilde{\mathbf{u}} \quad \text{(de tip Bilby)}$$

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Teoreme de descompunere a conexiunilor

(2) Masura de nemetricitate $\mathbf{Q}^e_{\mathcal{K}}$ a conexiunii elastice este data prin

$$\mathbf{Q}^e_{\mathcal{K}}\tilde{\mathbf{u}} = -\mathbf{C}^e(\mathbf{c}^p)^{-1}(\boldsymbol{\Lambda}\tilde{\mathbf{u}} \times \mathbf{I}) - [(\boldsymbol{\Lambda}\tilde{\mathbf{u}} \times \mathbf{I})]^T(\mathbf{c}^p)^{-1}\mathbf{C}^e. \quad (15)$$

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(3) Tensorul de torsiune de ordinul doi $\mathcal{N}^p_{\mathcal{K}}$ $(\equiv \mathcal{N}^e_{\mathcal{K}}$ $)$ este exprimat prin

$$\mathcal{N}^p_{\mathcal{K}} = \mathbf{F}^p \mathrm{curl}_{\mathcal{K}}(\mathbf{F}^p)^{-1} + (\mathbf{c}^p)^{-1}\Big(\mathrm{tr}\ \boldsymbol{\Lambda}\mathbf{I} - (\boldsymbol{\Lambda})^T\Big),$$

cu torsiunea Cartan $\mathbf{S}_{\mathcal{K}}$ and $\mathcal{N}^p_{\mathcal{K}}$ reprezentata prin (16)

$$(\mathbf{S}^p_{\mathcal{K}}\tilde{\mathbf{u}})\tilde{\mathbf{v}} = \mathcal{N}^p_{\mathcal{K}}(\tilde{\mathbf{u}} \times \tilde{\mathbf{v}}).$$

[4] Relatii cinematice:

- $$\mathbf{L} = \nabla_{\chi}\mathbf{v} \equiv \dot{\mathbf{F}}\mathbf{F}^{-1} \quad \text{gradientul vitezei} \tag{17}$$

$$\mathbf{L} = \dot{\mathbf{F}}^{e}(\mathbf{F}^{e})^{-1} + \mathbf{F}^{e}\underbrace{\mathbf{L}^{p}}(\mathbf{F}^{e})^{-1},$$

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- Vitezele de variatie ale conexiunilor & gradientul gradientului vitezei

$$\dot{\Gamma} = \mathbf{F}^{-1}((\nabla_\chi \mathbf{L})[\mathbf{F}, \mathbf{F}]$$

$$\frac{d}{dt}(\overset{(\mathrm{p})}{\boldsymbol{\mathcal{A}}_k}) = (\mathbf{F}^p)^{-1}((\nabla_{\mathcal{K}} \mathbf{L}^p)[\mathbf{F}^p, \mathbf{F}^p]. \tag{18}$$

II. Mecanica configurationala (sau materiala) :

4.1 Macro si micro forte

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Fortele materiale sau configurationale (de tip micro-tensiuni si micro-momente) satisfac ecuatiile de bilant ale microfortelor si sunt descrise constitutiv prin efecte disipative si nedisipative,

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sunt compatibile cu un principiu de disipare (formalizat prin conditia de nebilantare a energiei interne), ele avand semnificatie de forte conjugate prin putere cu deformatiile ireversibile de ordinul al doilea.

Cercetarea in domeniu poate fi imbogatita cu rezulatele datorate Sóos, Teodosiu, Iesan, Sandru din elasticitatea de tip Cosserat.

Macro forte

Macro-tensiunile , **forte fizice:**

$\mathbf{T}-$ tensorul de tensiune Cauchy nesimetric, $\boldsymbol{\mu}-$ macro- momente in χ.

1. Macro- Ecuatiile (locale) de bilant

$$\begin{aligned} \rho\mathbf{a} &= \text{div}\,\mathbf{T}^T + \rho\mathbf{b}_f, \quad \text{in} \quad \chi(\mathcal{P}, t) \\ \mathbf{T}^* &= \text{div}\,\boldsymbol{\mu} + \rho\mathbf{B}_m, \end{aligned} \tag{19}$$

$$\mathbf{T}^T\mathbf{n} = \mathbf{t}(\mathbf{n}), \quad \boldsymbol{\mu}\mathbf{n} = \mathbf{M}(\mathbf{n}) \quad \text{conditii pe frontiera } \partial\mathcal{P}_t.$$

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tensiunea (simetrica) Piola-Kirchhoff $\mathbf{\Pi}_{\mathcal{K}}$,

macro- momentele *pulled back* in $\mathcal{K}$ $\quad \boldsymbol{\mu}_{\mathcal{K}}$:

$$\mathbf{\Pi}_{\mathcal{K}} \equiv \mathbf{\Pi} = \text{det}\,(\mathbf{F}^e)(\mathbf{F}^e)^{-1}\mathbf{T}^s(\mathbf{F}^e)^{-T}, \quad \text{det}\mathbf{F}^e = \frac{\rho_{\mathcal{K}}}{\rho}$$

$$\boldsymbol{\mu}_{\mathcal{K}} = (\text{det}\,\mathbf{F}^e)(\mathbf{F}^e)^T \boldsymbol{\mu}[(\mathbf{F}^e)^{-T}, (\mathbf{F}^e)^{-T}].$$

Micro Ecuatii de bilant.

2. Micro- balance equation

$$\boldsymbol{\Upsilon}^p_{\mathcal{K}} = \text{div}\,(\boldsymbol{\mu}^p_{\mathcal{K}}) + \tilde{\rho}\mathbf{B}^p_m, \quad \text{in} \quad \mathcal{K}(\mathcal{P}, t),$$

$$\boldsymbol{\mu}^p_{\mathcal{K}}\mathbf{n} = \mathbf{M}^p(\mathbf{n}) \quad \text{on} \quad \partial\mathcal{K}(\mathcal{P}, t), \quad \text{micro- conditii de tractiune}\ . \tag{20}$$

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$\boldsymbol{\Upsilon}_{\mathcal{K}}^{p}-$ micro- tensiuni,

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tensiune de tip Mandel (nesimetrica) $\boldsymbol{\Sigma}_{\mathcal{K}} \equiv \Sigma$, in $\mathcal{K}$

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Ecuatiile de bilant pentru macro si micro forte, precum si conditiile pe frontiera au fost deduse din Principiul puterii virtuale in elasto-plasticitatea finita (cazul curburii plastice nule), formulat in S.C-T (2007) si sunt postulate pentru cazul general.

[5] Principiul de disipare

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$$-\dot{\psi}_{\mathcal{K}} + \frac{1}{2}\frac{\mathbf{\Pi}_{\mathcal{K}}}{\tilde{\rho}} \cdot \dot{\mathbf{C}}^e + \mathbf{\Upsilon}^p_{\mathcal{K}} \cdot \mathbf{L}^p + \frac{1}{\rho_{\mathcal{K}}}\boldsymbol{\mu}^p_{\mathcal{K}} \cdot \nabla_{\mathcal{K}}\mathbf{L}^p + \frac{1}{\rho_{\mathcal{K}}}\boldsymbol{\mu}_{\mathcal{K}} \cdot \underbrace{\mathcal{L}_{\mathbf{L}^p}[\overset{(e)}{\mathcal{A}}_{\mathcal{K}}]} + \frac{1}{\rho_{\mathcal{K}}}\boldsymbol{\mu}^{\lambda}_{\mathcal{K}} \cdot (\frac{d}{dt}(\mathbf{\Lambda}) \times \mathbf{I}) \geq 0, \tag{22}$$

care are loc $\mathbf{T}^* = -\mathbf{T}^a$.

$$\underbrace{\bullet} = (\mathbf{F}^e)^{-1}(\underbrace{\nabla_{\chi}\mathbf{L}})[\mathbf{F}^e, \mathbf{F}^e] - \underbrace{\nabla_{\mathcal{K}}\mathbf{L}^p}$$

5.1 Restrictii termomecanice: $\overset{(\cdot)}{\boldsymbol{\mathcal{A}}}$

Teorie de plasticitate de tip gradient corespunde cazului

$$\psi = \psi_{\mathcal{K}}(\mathbf{C}^e, \overset{(e)}{\boldsymbol{\mathcal{A}}}_{\mathcal{K}}, (\mathbf{F}^p)^{-1}, \overset{(p)}{\boldsymbol{\mathcal{A}}}_{\mathcal{K}}, (\boldsymbol{\Lambda} \times \mathbf{I})) \equiv$$

$$\overline{\psi}_k(\mathbf{C}, \boldsymbol{\Gamma}_k, \mathbf{F}^p, \overset{(p)}{\mathcal{A}}_k, \boldsymbol{\Lambda}. \times \mathbf{I}),$$

I. Relatii constitutive pentru macro-forte , de tip elastic exprimate prin potential $\psi_{\mathcal{K}}$

$$\frac{\boldsymbol{\Pi}_{\mathcal{K}}}{\rho_{\mathcal{K}}} = 2\, \partial_{\mathbf{C}^e} \psi_{\mathcal{K}}, \quad \frac{1}{\rho_{\mathcal{K}}} \boldsymbol{\mu}_{\mathcal{K}} = \partial_{\overset{(e)}{\boldsymbol{\mathcal{A}}}} \psi_{\mathcal{K}}.$$

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sau

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$$\frac{\mathbf{T}^s}{\rho} = 2\, \mathbf{F}\partial_{\mathbf{C}}\psi_k(\mathbf{F})^T, \quad \frac{1}{\rho_k}\boldsymbol{\mu}_k = \partial_{\boldsymbol{\Gamma}}\psi_k.$$

5.2 Ipoteze constitutive pentru micro-forte

- **III.** Micro-fortele contin

  (1) o parte disipativa

  (2) o componenta nedisipativa, microfortele energetice

$$\boldsymbol{\Upsilon}^p_{\mathcal{K}} = \rho_{\mathcal{K}}\Big(2\,\mathbf{C}^e\partial_{\mathbf{C}^e}\psi_{\mathcal{K}} + \partial_{\mathbf{F}^p}\overline{\psi}_k(\mathbf{F}^p)^T\Big) + \underbrace{Y_\gamma \mathbf{L}^p}$$

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- **II.** dissipation inequality

$$Y_\gamma \mathbf{L}^p \cdot \mathbf{L}^p + Y_\mu \nabla_{\mathcal{K}} \mathbf{L}^p \cdot \nabla_{\mathcal{K}} \mathbf{L}^p + Y_\lambda (\dot{\mathbf{\Lambda}} \times \mathbf{I}) \cdot (\dot{\mathbf{\Lambda}} \times \mathbf{I}) \geq 0. \quad (23)$$

  Macro si micro fortele satifac ecuatiile de bilant (??)

[6] Alte tipuri de modele:

de plasticitate cuplate cu zone deteriorate (tensorul metric neomogen
, descris prin ipoteza

existenta energiei libere in K de forma

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numai cu dislocatii continuu distribuite $(\tilde{\mathbf{S}}^{p}_{\mathcal{K}}\tilde{\mathbf{u}})\tilde{\mathbf{v}} = (\overset{(\mathrm{p})}{\mathcal{A}}_{\mathcal{K}}\ \tilde{\mathbf{u}})\tilde{\mathbf{v}} - (\overset{(\mathrm{p})}{\mathcal{A}}_{\mathcal{K}}\ \tilde{\mathbf{v}})\tilde{\mathbf{u}}$.

7. Perspective

Am dezvoltat o modelare matematica, capabila sa acopere domenii ale plasticitatii de ordinul doi, bazat pe configuratia cu torsiune

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Am dezvoltat o modelare matematica, capabila sa acopere domenii ale plasticitatii de ordinul doi, bazat pe configuratia cu torsiune

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Fortele materiale sau configurationale au fost definite ca fiind conjugate prin putere cu deformatiile ireversibile de ordinul al doilea, satisfac micro-ecuatiile de bilant si contin atat componente disipative cat si componente nedisipative.

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In perspectiva se va realiza studiul complet al ecuatiilor constitutive si a ecuatiilor de evolutie de tip conditii de curgere, compatibile cu disiparea si micro-ecuatiile de bilant.

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si tinand seama de prezenta neomogenitatilor.

Fortele materiale sau configurationale au fost definite ca fiind conjugate prin putere cu deformatiile ireversibile de ordinul al doilea, satisfac micro-ecuatiile de bilant si contin atat componente disipative cat si componente nedisipative.

In perspectiva se va realiza studiul complet al ecuatiilor constitutive si a ecuatiilor de evolutie de tip conditii de curgere, compatibile cu disiparea si micro-ecuatiile de bilant.

Se vor propune modele concrete elaborate in elasto- plasticitatea finita cu dislocatii continuu distribuite, sau cu zone deterioarare si se vor identifica functiile de material din datele de natura experimentala existente in literatura.