WORKSHOP EXPLORATORIU

DIRECTII ACTUALE SI DE PERSPECTIVA IN CERCETAREA MATEMATICA

17, 18 septembrie 2008
Bucuresti, Romania

PROGRAM
REZUMATE
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Workshop Exploratoriu

Directii actuale si de perspectiva in cercetarea matematica

17, 18 Septembrie 2008, Bucuresti, Romania

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Direcții actuale și de perspectivă în cercetarea matematică

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PROGRAM

Miercuri, 17 septembrie 2008

09:00–09:30	**Dan Burghelea**	Ceremonia de deschidere
09:30–10:20	**Daniel Tătaru**	Critical problems in nonlinear dispersive equations

Pauză de cafea

10:40–11:30	**Doina Cioră̆nescu**	Multi-scale structures and homogenization
11:40–12:30	**Liviu Ornea**	Locally conformally Kaeler geometry

Pauză de prânz

15:00–15:50	**Aurel Răşcanu**	Backward stochastic differential equations and financial models

Pauză de cafea

16:10–17:00	**Petru Mironescu**	Sobolev spaces of manifold valued maps
17:10–18:00	**Cristian Făciu, Sanda Cleja-Ţigoiu**	Research trends in solid mechanics: tradition and novelty

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09:30–10:20	**Gabriel Turinici**	Quantum chemistry and control: theoretical, experimental and numerical challenges

Pauză de cafea

10:40–11:30	**Sebastian Aniţa**	Internal nonnegative stabilization for some parabolic equations
11:40–12:30	**Camil Muscalu**	On a new multi-parameter structure in harmonic analysis and its connections to the theory of differential equations

Pauză de prânz

15:00–15:50	**Paltin Ionescu**	On manifolds covered by lines

Pauză de cafea

16:10–17:00	**Florin Rădulescu**	Applications of Operator Algebra in Number Theory
17:10–18:00	**Marius Tucsnak**	Ingham-Beurling inequalities, number theory and control of PDE's

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REZUMATE

Internal nonnegative stabilization for some parabolic equations

Sebastian Aniţa

The internal zero-stabilization of the nonnegative solutions to some parabolic equations is investigated. We provide a necessary and a sufficient condition for nonnegative stabilizability in terms of the sign of the principal eigenvalue of a certain elliptic operator. This principal eigenvalue is related to the rate of the convergence of the solution. We give some evaluations of this principal eigenvalue with respect to the geometry of the domain and of the support of the control. A stabilization result for an age-dependent population dynamics with diffusion is also established.

AMS Subject Classification: 35K05, 35P15, 93D15, 92D25, 35B37

Keywords: Nonnegative stabilization, parabolic equations, Rayleigh's principle, comparison principle, age-dependent population dynamics

Multi-scale structures and homogenization

Doina Ciorănescu

We give here some applications of the periodic unfolding method ([3] and [7]), which is well suited to approach dierent classes of periodic homogenization problems. In particular, we present its applications to multi-scale homogenization as well as to the homogenization of some nonlinear problems.

The convergence result was established in [1] and [2] by sophisticated -convergence arguments. Since then, many attempts at simplifying the proof (for example by using two-scale convergence), seem not to have borne fruit.

In [4], a joint paper with Alain Damlamian and Riccardo De Arcangelis, we apply the tool of periodic unfolding from [3]. This gives a direct proof of the convergence result (under slightly weaker assumptions than in [1] and [2]), making use of simple weak convergence arguments in L^p-type spaces. This paper is part of a series of ongoing works concerning the applications of the periodic unfolding method to homogenization. The second one [5], treated the same problem but in the case of convex densities. The third one [6], that will be presented here, deals with constrained integral type energies.

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Research trends in solid mechanics: tradition and novelty

Cristian Făciu & Sanda Cleja-Ţigoiu

The continuum mechanics, physical and experimental approaches together with computational mechanics are required to advance the knowledge in material science and to contribute to solve complex problems occurring in real industrial practice. In the past, until the 1980's the plasticity with applications to metal forming (S. Cleja-Ţigoiu, N. Cristescu, Ed. Univ. Bucureşti -1985) has been developed within the classical theory of plasticity (small deformation rate-independent or rigid/viscoplastic models). At present, large deformation formalism within a macroscopic approach to elasto-plasticity (at which we refer here in the first part) is developed in order to describe the irreversible behaviour of materials. A first step in an axiomatic reconstruction of the models with internal state variables and relaxed configurations in finite plasticity has been done by Cleja-Ţigoiu and Soós [1], based mainly on Teodosius approaches (1970, 1975). The related topics to mathematical description of the models, like (initial and developed) anisotropy, dissipative nature of the plastic deformation, role of the plastic spin, bifurcation of the solution in crystal plasticity, variational inequalities associated with rate boundary value problems at a generic stage of elasto-plastic processes, are formalized and analyzed in a set of papers (see for instance [2]-[4]). On the other hand, the material or configurational Mechanics experienced a remarkable revival over the last two decades in order to capture the material inhomogeneities, for describing the structural changes in continua, with the goal to unify treatment of different phenomena like fracture and damage evolution, plasticity and dislocation motions. The research project "Irreversible behaviour of elasto-plastic materials with structural inhomogeneities like dislocations and twinned structures", awarded by Fulbright Program and started at Texas A&M University in collaboration with K. Rajagopal, has been continued by Cleja-Ţigoiu in [5]. A general mathematical framework, able to cover a large range of second order plasticity (see also [6]), based on anholonomic configuration, taking into account the presence of the inhomogeneities such as continuously distributed dislocations, has been developed in [5]. Material forces (stress and stress momentum) satisfy the micro-balance equations and contain dissipative and non-dissipative parts, being compatible with the free energy imbalance and they are driving forces on second order pair of irreversible deformations. An appropriate connection with Romanian researches (promoted by Soos, Teodosiu, Iesan, Sandru) in the field of Cosserat approach to elasticity could be done.

In the second part we discuss different aspects related to the phenomenological modelling of solid deforming bodies by using rate-type constitutive equations. Romanian school of mechanics, through the research works of N. Cristescu, I. Suliciu (see for instance [7], [8]) and their collaborators, has promoted and studied intensively starting with the 60 such theories. It was a significant effort to include time effects in constitutive equations in order to describe new interesting phenomena. Thus, elastic-viscoplastic models have been applied to various materials such as metals, soils, rocks and polymers for both quasi-static and dynamic problems. Moreover, mathematical models based on rate-type constitutive equations have been used by I. Suliciu to describe the corona losses in electric-transmission lines . A new direction (see [9-10]) has been developed in the last 20 years and it concerns the capacity of such constitutive relations to describe instability phenomena which accompany phase transformations in smart materials like shape memory alloys. From thermodynamical point of view this approach is related with the use of non-convex free energy functions, while from mathematical point of view it is related to the approximation of the solutions of a mixed hyperbolic-elliptic partial differential equations system with those of a hyperbolic one. Since a direct phase transformation is exothermic and the reverse one is endothermic a new step has been done in [11] by including thermal effects in a rate-type constitutive description. We illustrate here how the spontaneous nucleation and phase transformation of narrow zones along a tensile specimen is accompanied by a local self heating. Recently, dynamic aspects of solid-solid phase transitions have been investigated (see [12]). We have proposed longitudinal impact experiments of thin bars as an effective mean for understanding the kinetics of stress-induced phase transformations in shape memory alloys (SMA). By using Hadamards theory of wave propagation one has obtained important insight into the wave structure. Mathematical aspects related with the non-uniqueness of the solutions for Riemann and Goursat problems are discussed. The predictions of the rate-type model are analyzed and illustrated by numerical results.

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On manifolds covered by lines

Paltin Ionescu

Algebraic manifolds embedded in some projective space have a very rich interaction between their intrinsic and extrinsic geometry. Much of it is related to LINES in the ambient projective space (e.g. tangents, secants, etc.). Although very special, manifolds COVERED by lines (sometimes also called "ruled"), have been studied for a long time and still are a source of fascinating open questions. I will try to survey some recent progress on this classical theme of projective geometry, mostly centering around the concept of "DEFECTIVE manifold". Somewhat surprisingly, at least for the non-expert, to treat such simple curves as lines, we need rather sophisticated modern techniques.

Sobolev spaces of manifold valued maps

Petru Mironescu

Let M, N be compact manifolds; M may have a boundary, but not N. For $0 < s < \infty$ and $1 \leq p < \infty$, we consider the metric space $X_{s,p} = \{u : M \to N \ ; \ u \in W^{s,p}\}$. Here, $W^{s,p}$ is a Sobolev space.

Such spaces appear naturally in geometry or PDEs inspired by physics (liquid crystals, superconductivity, micromagnetism). The problems we will address go beyond the special cases of interest to applications.

Question 1: describe the connected components of $X_{s,p}$.
Question 2: decide whether smooth maps are dense in $X_{s,p}$.
Question 3: if the answer to Question 2 is negative, find a class of "as smooth as possible" maps dense in $X_{s,p}$.
Question 4: again, if the answer to Question 2 is negative, describe the closure of smooth maps in $X_{s,p}$.
Question 5: if M has a boundary, describe the traces on ∂M of maps in $X_{s,p}$.

In full generality (each s, p, M and N), the answers are not known. We will discuss the main results: the work of Bethuel in the early 90's (on Questions 2-4), the series of papers of Hang and Lin (on Questions 1-3), and the very recent solution to Questions 1-5 in the very special case $N = \mathbb{S}^1$.

We will end by discussing the reasonable progress expected at short term and reasonable strategies to attack these questions.

On a new multi-parameter structure in harmonic analysis and its connections to the theory of differential equations

Camil Muscalu

We shall describe (at least) two quite distinct problems coming from the general theory of differential equations and show that their analysis is deeply related to the understanding of some very interesting multi-parameter objects.

Locally conformally Kaehler geometry

Liviu Ornea

I shall present recent results on the geometry and topology of locally conformally Kaehler manifolds, stressing the common points and the dichotomy with Kaehler geometry. I shall focus on embedding properties and on the existence of global potential functions.

Applications of Operator Algebra in Number Theory

Florin Rădulescu

The analysis of discrete group actions as initiated by A Connes in the Noncommutative Geometry framework is appled to the study of the spectral gap for Hecke operators acting on Maass waveforms.

Backward stochastic differential equations and financial models

Aurel Răşcanu

In 1973, Fischer Black and Myron Scholes published their ground-breaking pioneering paper The Pricing of Options and Corporate Liabilities. Not only did this paper specify the first successful option pricing formula, the so-called Black-Scholes formula, but it also described a general framework for pricing other derivative instruments. That paper launched the field of financial engineering and has been, together with the technological progress (informatics, telecommunications) at the origin of the quasi frenzied development of new finance markets (the first, the CBOT, opened in Chicago in 1973, quickly followed by others, first in the USA (Philadelphia,...), and then at many other places in the world and also in Europe). However, the application of the Black-Scholes formula for the pricing of options requires to determine the unknown volatility in the underlying asset. The current practice which has developed in the markets is to calculate the volatility by reversing the Black-Scholes formula in which the observed market price for the option is substituted. If the chosen model of the underlying asset were adequate this implicit volatility would coincide with the one entering in the Black-Scholes formula, but in practice it turns out that the implicit volatility becomesrandom. A first approach to solve the problems related with was to use the robustness of the Black-Scholes formula in order to deduce from the existence of deterministic bounds for the implicit volatility the corresponding bounds of the option price process. However, recent turbulences in financial markets (the crash of 1987 and the Asian crisis of 1997, but surely also the actual heavy price fluctuations linked with the mortgage lender crises in the USA) have made the agents on the financial markets and the users more and more sensitive to the intrinsic risk of the management of options and other derived products. To improve the Black-Scholes and volatility models different approaches have been developed. If the volatility parameter is deterministic or a deterministic function of the underlying asset price, the Black-Scholes model and its generalizations (which, for instance, study dividend payment, transaction costs,...) lead to backward stochastic differential equations whose driver is a deterministic function of the solution (composed of the option price and the portfolio processes) and the underlying asset process. When the portfolio value process or, for instance, the interest rates is forced to stay in a prescribed set it is arrived to viability problems or stochastic variational inequalities.

Critical problems in nonlinear dispersive equations

Daniel Tătaru

Nonlinear dispersive equations model wave-like phenomena where nonlinear interactions occur. In a broad sense, critical means that the effect of the nonlinear interactions is comparable to the effect of the linear evolution. The aim of the talk is to introduce some of the more interesting problems in this direction, and to describe recent results and open problems concerning global regularity vs. blow-up phenomena.

Ingham-Beurling inequalities, number theory and control of PDE's

Marius Tucsnak

We present effective versions of a classical inequalities on non-harmonic Fourier series due to Ingham and Beurling. Combined to results from analytic number theory (some of them being new), we provide necessary and sufficient for the exact observability of systems governed by the Schrödinger equation in a rectangle with Dirichlet or Neumann boundary conditions. Generalizing results from Ramdani, Takahashi, Tenenbaum and Tucsnak [1], we prove that the corresponding criterion is that the *observation region has non empty interior* in the case of Dirichlet observation and, in the case of Neumann observation, that it has an open intersection with an edge of each direction. Thus, in both circumstances, observation regions may have *arbitrarily small measures*. We complement these results by proving that the above mentioned properties hold *in arbitrarily small time*. We also show that similar results hold for the Euler-Bernoulli plate equation. Finally, we give explicit estimates for the blow-up rate of the observability constants as the time and (or) the size of the observation region tend to zero.

For details on the presented results we refer to [2] and [3].

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Quantum chemistry and control: theoretical, experimental and numerical challenges

Gabriel Turinici

Chemistry is the basis of many industrial (e.g. combustion), biological (e.g. photosynthesis) and life sciences (e.g. biophysics) applications. After a brief overview of these domains, we will focus the optical manipulation of quantum phenomena. As an illustration of the interactions within mathematicians, chemists and physicists, we will address in this talk the present state of the art in quantum control (both at the theoretical and numerical level) and explain how this impacts the experimental research. Part of the work builds on a collaboration with **C. Lefter** and **A. Zălinescu** from the Romanian Academy (at Iasi).